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Open Geospatial Consortium

TESTBED-18: 3D+ STANDARDS FRAMEWORK ENGINEERING REPORT

ENGINEERING REPORT

PUBLISHED

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Editor: Frieder Schmid, Mohammad J. Tourian, Charles Heazel, Nico Sneeuw

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I EXECUTIVE SUMMARY

The OGC Testbed 18 Engineering Report (ER) begins with an overview of important Standards Development Organizations and authorities whose Standards are discussed in this ER. Current Standards from ISO, such as *ISO 19111: Geographic Information – Spatial referencing by coordinates*, and the OGC GeoPOSE Standard, are not adequate for dealing with non-Earth geospatial data.

Alternative approaches by geodetic and astronautic organizations, such as the Consultative Committee for Space Data Systems (CCSDS) *Navigation Data — Definitions and Conventions*, the International Earth Rotation and Reference Systems Service (IERS) *IERS conventions*, or the NASA NAIF *SPICE Toolkit* are also being presented.

The resulting practices are often similar to each other but cannot be understood as defining a Standard framework or approach. In describing the coordinate reference systems and frames used in geodesy, the inadequacies and weaknesses of the existing Standards become apparent. In general, from a geodetic point of view, the main classes of coordinate reference systems (CRS) are as follows:

- Space-fixed systems,
- Earth-fixed systems,
- Local systems.

The exact distinction of these CRS is important for understanding transformations between them. Therefore, this ER describes these transformations. The detailed discussion includes the effect of precession, nutation, and polar motion. In addition, this ER emphasizes the need for taking time systems into account in the definition of coordinate systems and their consideration in the development of Standards. This ER specifically describes the following time systems.

- Sidereal time, which refers to the stars
- Solar or universal time, which refers to the Sun
- Atomic time, which refers to atomic phenomena
- Theoretical time, which refers to a theoretical model

Furthermore, the above theoretical discussion is supported by a demonstration in which two objects are represented in different coordinate systems. The demonstration illustrates the difference between space-fixed and Earth-fixed reference systems. Finally, this ER provides an evaluation of the existing Standards from a geodetic perspective and makes recommendations for the identified deficiencies. These should be considered when expanding current Standards. The range of objects that can be represented in the extension of Standards can be expanded enormously. This extension approaches the goal of forming a general framework for 3D+

coordinate systems to allow a complete, unambiguous, and universally understood state of any object in space.

II KEYWORDS

The following are keywords to be used by search engines and document catalogues.

testbed-18, Inertial Reference System, Terrestrial Reference System, Coordinate Reference System, Time system

III SECURITY CONSIDERATIONS

No security considerations have been made for this document.

IV SUBMITTING ORGANIZATIONS

The following organizations submitted this Document to the Open Geospatial Consortium (OGC):

- Institute of Geodesy, University of Stuttgart

V ABSTRACT

Currently, most OGC Standards focus on data that is observed on the ground or near the Earth's surface. Extra-terrestrial space and the exact location of remote sensors has been less in focus. Current OGC Standardizations cannot be applied to this type of spatial data processing. This OGC Testbed 18 Engineering Report (ER) first provides a detailed description of existing Standards, conventions, and tools which are particularly relevant for further evaluation. Subsequently, various coordinate and time systems are presented and improvements or extensions to existing Standards are proposed to describe objects in orbit around any celestial body or interplanetary flight through our solar system.

1 SCOPE

1 SCOPE

This Testbed 18 Engineering Report provides information about various coordinate systems used in geodesy and aerospace. This ER is applicable to existing Standards and can be used by OGC Standard Working Groups (SWGs) as a source of ideas and recommendations for future evolution of relevant Standards.

2

NORMATIVE REFERENCES

2 NORMATIVE REFERENCES

The following documents are referred to in the text in such a way that some or all of their content constitutes requirements of this document. For dated references, only the edition cited applies. For undated references, the latest edition of the referenced document (including any amendments) applies.

Open API Initiative: **OpenAPI Specification 3.0.2**, 2018 https://github.com/OAI/OpenAPI-Specification/blob/master/versions/3.0.2.md

], ISO 19107:2003 Geographic information — Spatial schema, 2003 https://www.iso.org/standard/26012.html

ISO 19108:2002 Geographic information — Temporal schema, 2002 https://www.iso.org/standard/26013.html

ISO 19109:2015 Geographic information — Rules for application schema, 2015 https://www.iso.org/standard/59193.html

ISO 19111:2019 Geographic information — Referencing by coordinates, 2019, https://www.iso.org/standard/74039.html

ISO 19141:2008 Geographic information — Schema for moving features, 2008 https://www.iso.org/standard/41445.html

Heazel, C., Kolbe, T., Kutzner, T., Nagel, C., Roensdorf, C., Smyth, C. S.: OGC 20-010, **OGC City Geography Markup Language (CityGML) Part 1: Conceptual Model Standard**, 2021 https://docs.ogc.org/is/20-010/20-010.html

Kim, K., Ishimaru, N.: OGC 19-045r3, **OGC Moving Features Encoding Extension — JSON**, 2020 http://docs.opengeospatial.org/is/19-045r3/19-045r3.html

R. Fielding, J. Gettys, J. Mogul, H. Frystyk, L. Masinter, P. Leach, T. Berners-Lee: IETF RFC 2616, *Hypertext Transfer Protocol — HTTP/1.1*. RFC Publisher (1999). https://www.rfc-editor.org/info/rfc2616.

ISO: **Geographic information — Reference model — Part 1: Fundamentals**, 2014. https://www.iso.org/standard/59164.html

OGC: **GeoPose Specification draft**, 2021. https://github.com/opengeospatial/GeoPose/

3

TERMS, DEFINITIONS AND ABBREVIATED TERMS

3 TERMS, DEFINITIONS AND ABBREVIATED TERMS

This document uses the terms defined in OGC Policy Directive 49, which is based on the ISO/IEC Directives, Part 2, Rules for the structure and drafting of International Standards. In particular, the word “shall” (not “must”) is the verb form used to indicate a requirement to be strictly followed to conform to this document and OGC documents do not use the equivalent phrases in the ISO/IEC Directives, Part 2.

This document also uses terms defined in the OGC Standard for Modular specifications (OGC 08-131r3), also known as the ‘ModSpec’. The definitions of terms such as standard, specification, requirement, and conformance test are provided in the ModSpec.

For the purposes of this document, the following additional terms and definitions apply.

3.1. coordinate

One of a sequence of numbers designating the position of a point.

Note 1 to entry: In a spatial coordinate reference system, the coordinate numbers are qualified by units.

[**SOURCE:** ISO 19111]

3.2. coordinate system

A set of mathematical rules for specifying how coordinates are to be assigned to points.

[**SOURCE:** ISO 19111]

3.3. coordinate reference system

A coordinate system that is related to an object by a datum.

Note 1 to entry: Geodetic and vertical datums are referred to as reference frames.

Note 2 to entry: For geodetic and vertical reference frames, the object will be the Earth. In planetary applications, geodetic and vertical reference frames may be applied to other celestial bodies.

[**SOURCE:** ISO 19111]

3.4. Abbreviated terms

CCSDS	Consultative Committee for Space Data Systems
CEP	Celestial Ephermis Pole
GNSS	Global Navigation Satellite Service
GPS	Global Positioning Service
IERS	International Earth Rotation and Reference Systems Service
ISO	International Organization for Standardization
NAIF	Navigation and Ancillary Information Facility
NCP	North Celestial Pole
NEP	North Ecliptical Pole
$\mathbf{R}_i(\alpha)$	Rotation around axis i with angle α
UTC	Universal Time Coordinated

4

INTRODUCTION

4 INTRODUCTION

The exact positioning of sensors in 3D space and corresponding 3D data streaming, analytics, and portrayal plays an important role in many geospatial scenarios and applications. Remote sensing of the Earth's ground, atmosphere, or stratosphere has become routine in many domains.

In classical geodesy, a Coordinate Reference System (CRS) is a framework used to precisely measure locations on the surface of the Earth as coordinates. Defining a CRS is the application of the abstract mathematics of coordinate systems and analytic geometry to geographic space. The definition of a specific CRS comprises a choice of Earth ellipsoid, horizontal datum, map projection (except in the geographic coordinate system), origin point, and unit of measure. Thousands of CRSs have been specified for use around the world or in specific regions and for various purposes, necessitating transformations between different CRSs. CRSs are a crucial basis for the sciences and technologies of Geoinformatics, including cartography, geographic information systems, surveying, remote sensing, and civil engineering. This has led to international Standards such as the *OGC-ISO 19111:2019 Geographic information—Spatial Referencing by coordinates*.

The TestBed-18 activity documented in this ER aims to go beyond the surface of the Earth and enable full location determination, orientation, and trajectory description of objects in orbit around celestial bodies or in free flight in our solar system. This ER evaluates current Standards with respect to the exact positioning of sensors at any location within the solar system. This ER proposes extensions to current Standards to cover broader needs such as an inertial coordinate reference system.

5

STANDARDS, CONVENTIONS AND TOOLS

5 STANDARDS, CONVENTIONS AND TOOLS

5.1. Authorities responsible for providing Standards and definitions for coordinate reference systems

A complete listing of all organizations contributing to this topic would exceed the scope of this report. Therefore only the most relevant are presented in this ER.

5.1.1. International Organization for Standardization

The International Organisation for Standardization (ISO) is a global network of 167 national Standards bodies with one member per country. It is an independent, non-governmental organization that provides a platform for developing practical tools through common understanding and cooperation with all stakeholders. ISO develops, approves and publishes Standards for everything except electrical and electronic engineering and telecommunication.

NOTEElectrical and electronic engineering Standards are developed by the International Electrotechnical Commission (IEC), and the telecommunication by the International Telecommunication Union (ITU).

5.1.2. Open Geospatial Consortium

The Open Geospatial Consortium (OGC) is an open-membership, worldwide voluntary consensus Standards organization that defines, documents, approves, and maintains geoprocessing Standards. OGC geospatial Standards can be used in geographic information systems (GIS) and systems for Earth imaging, Web mapping, location-based services, surveying and mapping, CAD-based facility management, webs of geolocated sensors, navigation, cartography, automated mapping, etc. The OGC Standards define open interfaces, protocols, schemas, and other components. Implementation of these Standards enable different systems and applications to exchange geospatial data and instructions and enable the complete integration of these capabilities into a range of information systems. The OGC collaborates with other Standards organizations such as the W3C and ISO. Since several Standards were jointly developed by ISO and the OGC they will be discussed together.

5.1.3. Consultative Committee for Space Data Systems

Another organization that often cooperates with ISO is the Consultative Committee for Space Data Systems (CCSDS). CCSDS was formed by the major space agencies of the world to provide a forum for discussion of common problems in the development and operation of space data systems. CCSDS is currently composed of 11 member agencies, 32 observer agencies, and over 119 industrial associates. CCSDS Standards are widely used in aerospace. The publications of the CCSDS can be divided into different categories, color coded. The most important are Recommended Standards (Blue), Recommended Practices (Magenta), and Informational Reports (Green). Therefore, the CCSDS does not necessarily provide Standards but rather conventions and practices.

5.1.4. International Earth Rotation and Reference Systems Service

The main objective of the International Earth Rotation and Reference Systems Service (IERS) is the suggested conventions regarding various transformations and reference system definitions that are widely used in geodesy and of great importance for the work defined in this ER. The IERS provides definitions of various coordinate reference systems and their realizations as well as the Earth orientation parameters required to study Earth orientation variations to transform between the defined CRSs. For all systems and frames, the associated Standards, constants and models are given.

A detailed discussion of the IERS observations, conventions, and more is made in Clause 6 — Clause 8.

5.1.5. NASA's Navigation and Ancillary Information Facility

NASA's Navigation and Ancillary Information Facility is responsible for developing a tool that will be discussed later in this ER. NASA's Navigation and Ancillary Information Facility (NAIF) was established at the Jet Propulsion Laboratory to lead the design and implementation of the "SPICE" ancillary information system. The NAIF is dedicated to the issues of producing high precision, clearly documented and readily used "ancillary information" required by space scientists and engineers.

5.1.6. International Astronomical Union

The International Astronomical Union (IAU) was founded in 1919 to promote and safeguard the science of astronomy in all its aspects, including research, communication, education, and development, through international cooperation. Its members — structured into Divisions, Commissions, and Working Groups — are 12113 astronomers active in professional research and education in astronomy from 93 countries worldwide. Among other activities, it acts as the recognized authority for assigning designations and names to celestial bodies (stars, planets, asteroids, etc.) and any surface features on them.

The Working Group on Cartographic Coordinates and Rotational Elements was established with the purpose to avoid a proliferation of inconsistent cartographic and rotational systems, and therefore to define the cartographic and rotational elements of the planets and satellites on a systematic basis and to relate the new cartographic coordinates rigorously to the rotational elements.

5.2. ISO Standardizations

The first and most important set of standards referenced in this ER are standards developed by ISO Technical Committee 211 (TC211) and the Open Geospatial Consortium (OGC). These Standards define foundational concepts which will serve as the basis for later discussions, evaluations, and requirements.

5.2.1. Referencing by coordinates

ISO 19111:2019 Geographic information — Referencing by coordinates is arguably the most important standard addressed by this initiative. TC211 and OGC strive to maintain a separation between coordinate values and the coordinate reference system where those values apply. This allows new coordinate reference systems (projections, geoids, etc.) to be defined without impacting existing standards and implementations. However, this requires that there be a standard way to describe a coordinate reference system. ISO 19111 fills this need by defining "the conceptual schema for the description of referencing by coordinates" and the data required to define a coordinate reference system. ISO 19111 compliant registries allow applications to retrieve, interpret, and apply these coordinate reference system definitions at run time. This is a key capability enabling geospatial technologies to be applied to non-Earth locations.

A detailed description of this ISO 19111 can be found in the OGC Testbed 18 D025 — *Reference Frame Transformation Engineering Report* (OGC 22-038) and is therefore not discussed here.

5.2.2. Temporal Schema

ISO 19108:2002 Geographic information — Temporal schema defines concepts for describing temporal characteristics of geographic information. The majority of this Standard addresses the representation of Earth-centric systems (calendars) for dates. However, it also provides a framework for representing time. This framework allows for the association of spatial and temporal reference systems. These associations make it possible to define a spatial-temporal reference system; a coordinate reference system for 4D space-time. This concept is explored in greater detail within this Engineering Report and in D025 — *Reference Frame Transformation Engineering Report*.

5.2.3. A Feature Model for Space Objects

Before we can describe non-terrestrial objects, we must first have a conceptual model of the universe. The OGC and ISO TC211 provide that model in ISO 19109:2015 Geographic information — Rules for application schema. 19109 defines the General Feature Model (GFM). The GFM defines the concept of a Feature, its components, and behaviors.

The OGC and ISO define a Feature as an "Abstraction of real-world phenomena" (ISO 19101-1:2014).

A Feature, then, is a high-level abstraction for anything that does or could exist in the universe.

The General Feature Model further refines the concept of a Feature. The following principles are most relevant for this ER.

1. A Feature can be a FeatureType or an Instance of a FeatureType (AnyFeature).
2. FeatureTypes can form a taxonomy (inheritance).
3. Features possess characteristics (Properties).
4. A Property of a Feature can be an Operation, Attribute, or Association.

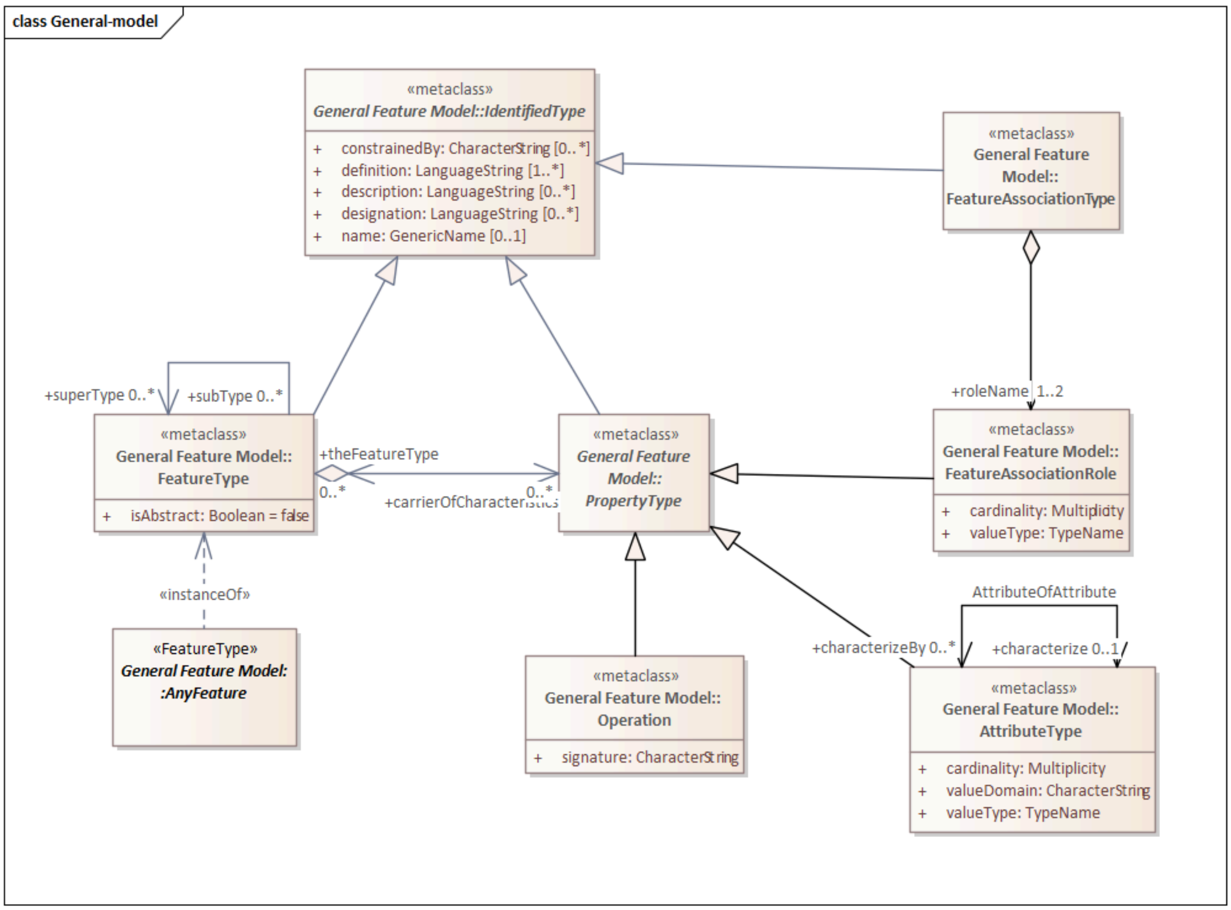

Figure 1 — General Feature Model

The resulting model is sufficient to describe a Feature's identity (IdentifiedType), what it is (FeatureType), what it can do (Operations), the Feature's observable characteristics (Attributes), and any associations with other Feature instances.

5.2.4. Geometry in 3 Dimensions

The applicable OGC/ISO standard for geometries is ISO 19107:2003 Geographic information — Spatial schema. While a new version was approved in 2019, it hasn't propagated through the rest of the ISO standards baseline. As a result, 19107:2003 is the most recent implemented version.

5.2.4.1. Features and Geometry

The General Feature Model treats geometry as an attribute of the Feature. In addition, it defines the following three types of attributes which are useful for associating geometry with a Feature in a standard manner.

- SpatialAttributeType: Geometries (GM_Object) and Topologies (TP_Object)
- LocationalAttributeType: Named locations, extents, and points
- TemporalAttributeType: Temporal objects (TM_Object)

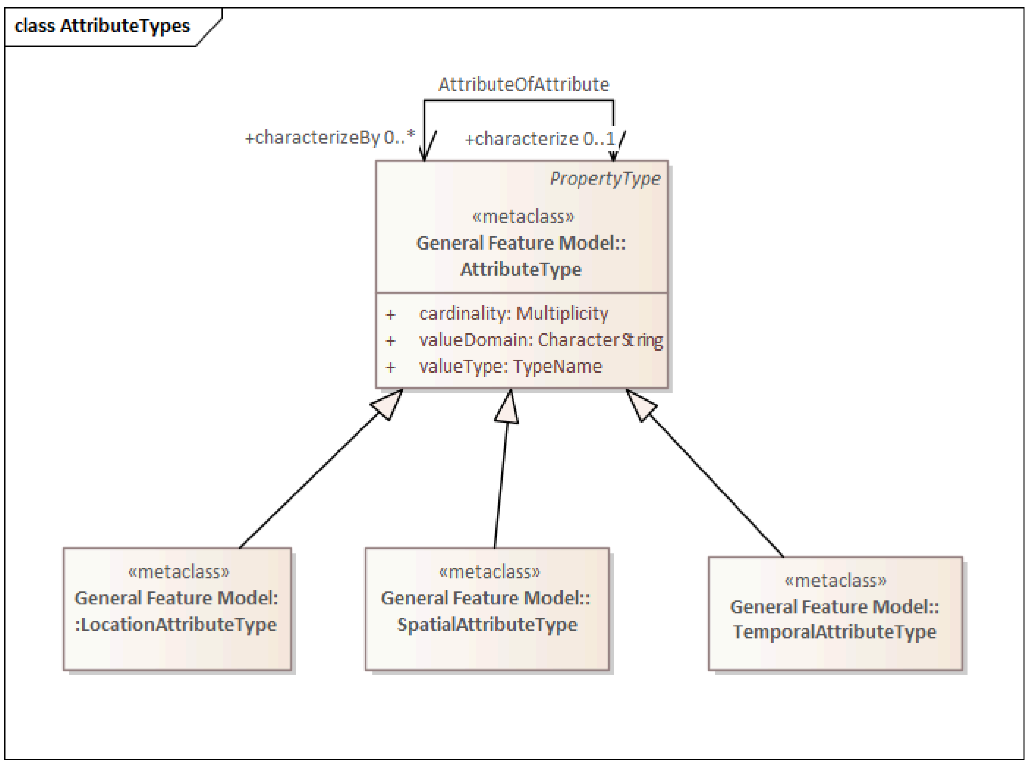

Figure 2 — General Feature Model Attribute Types

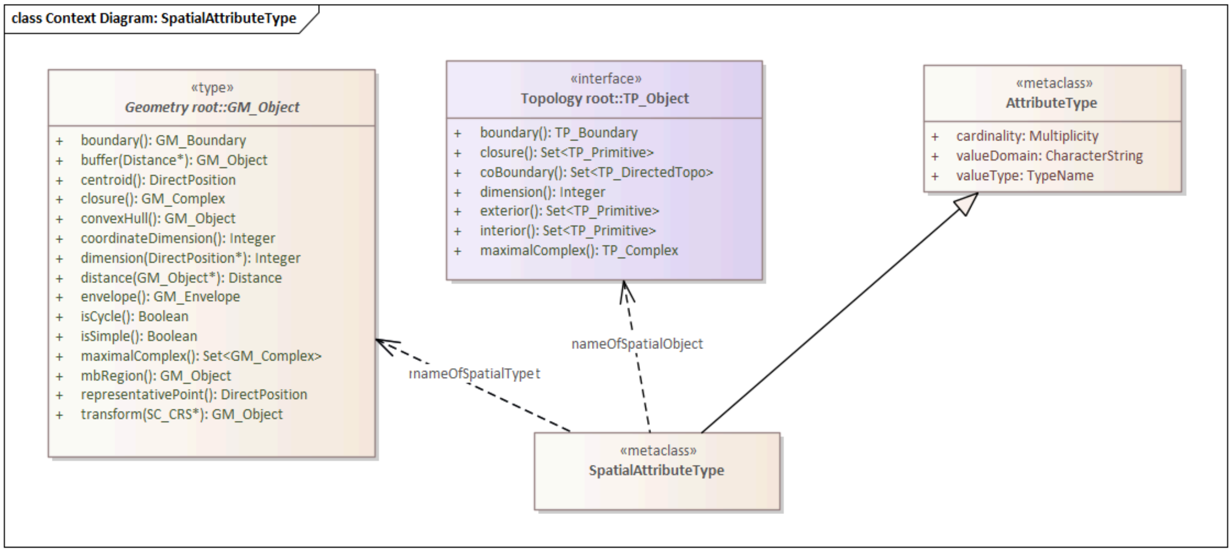

Figure 3 — General Feature Model Spatial Attribute Types

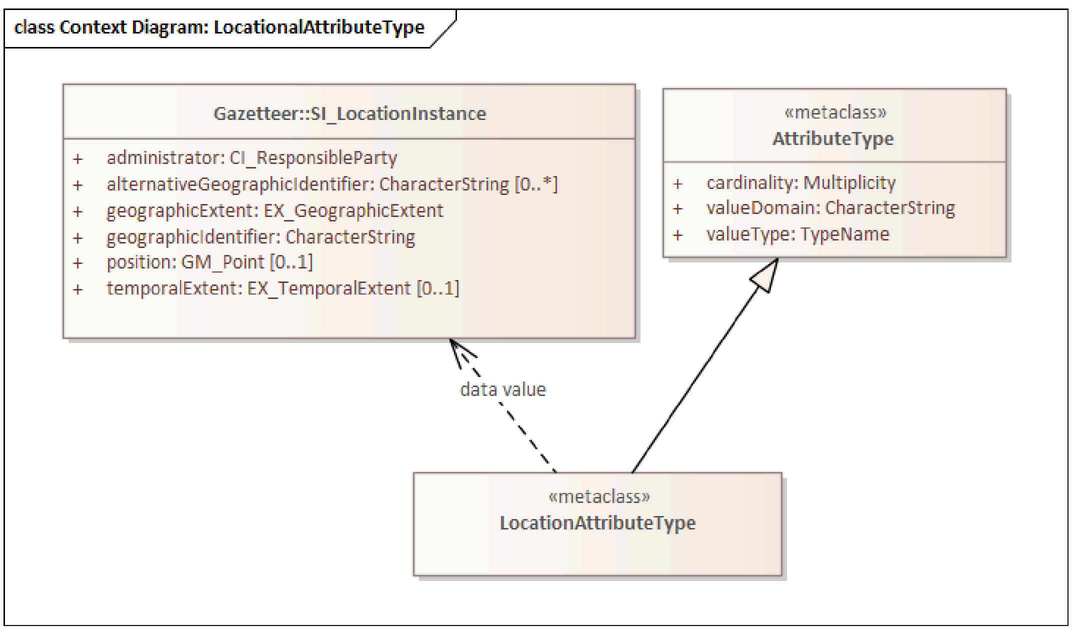

Figure 4 — General Feature Model Locational Attribute Types

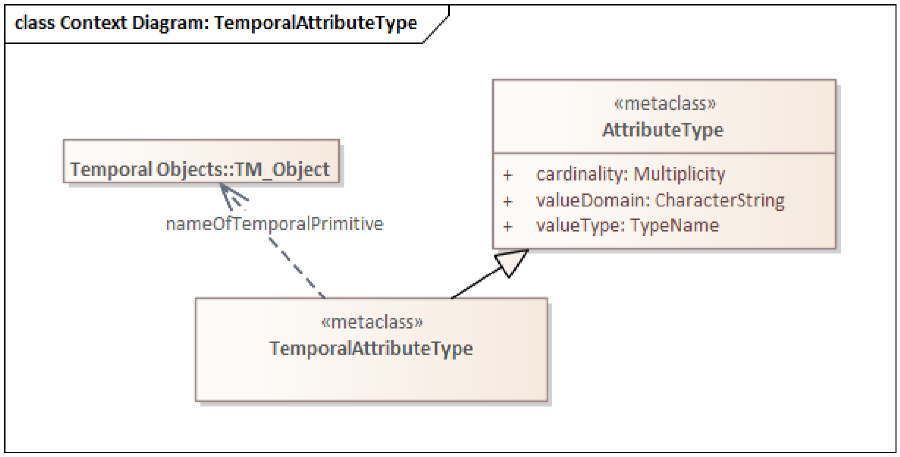

Figure 5 — General Feature Model Temporal Attribute Types

5.2.4.2. The Geometry Model

An important feature of the ISO Geometry Model is that it does not specify or assume a coordinate reference system. The SC_CRS class, defined in ISO 19111, is used to define a coordinate reference system. The “Coordinate Reference System” association is used to associate a GM_Object instance with the appropriate SC_CRS instance. Since all geometry classes are descended from GM_Object, any geometry object can have its own unique coordinate reference system.

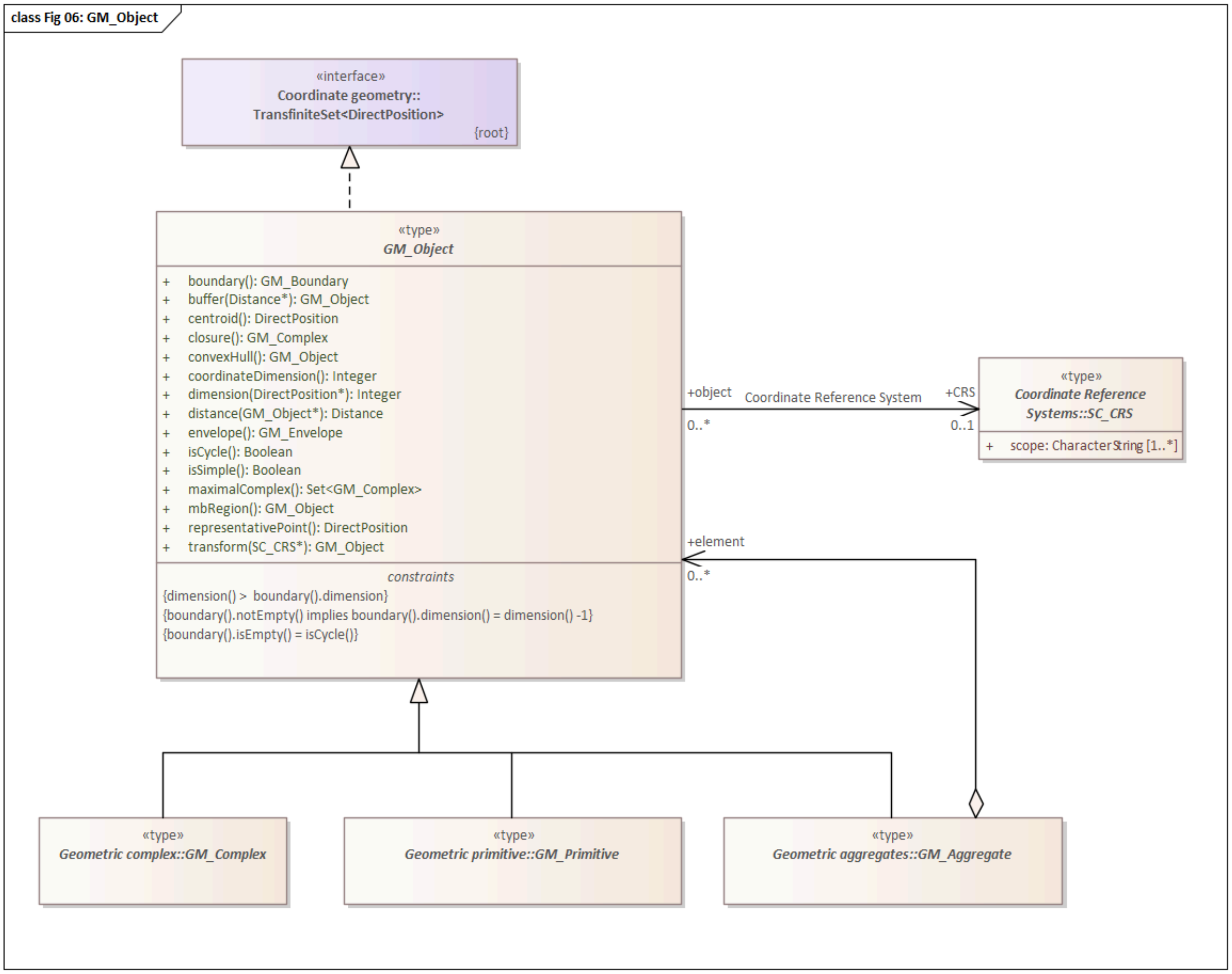

Figure 6 — GM_Object UML Model

While the ISO Geometry Model is very complex, at its core is the DirectPosition class. This is the fundamental specification of a position within a coordinate reference system. Its purpose is simply to hold the coordinates for a position within the specified coordinate reference system.

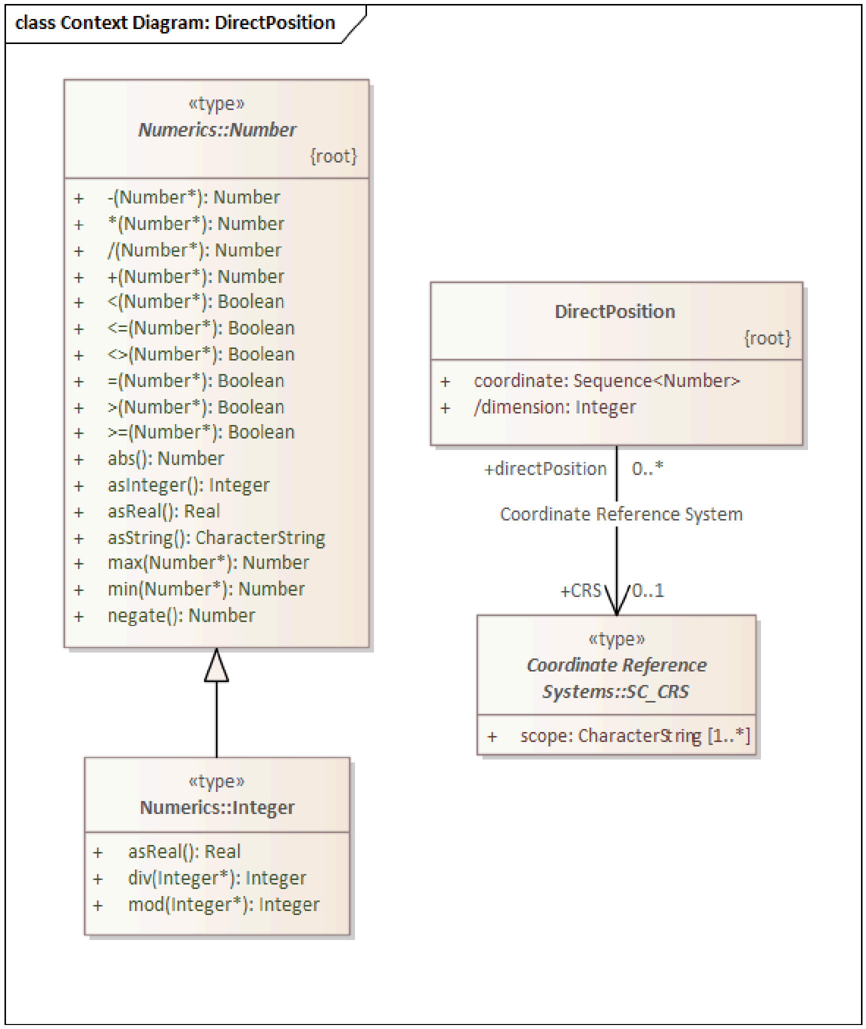

Figure 7 — Direct Position UML Model

The DirectPosition class contains two attributes, “dimension” and “coordinate”. The “coordinate” attribute is a sequence of numbers. Each number represents the location of the DirectPosition on a coordinate axis. There are no constraints on the number of numbers in the sequence. Therefore, a DirectPosition can represent an unlimited number of dimensions. The “dimension” attribute specifies the number of axis in the applicable coordinate reference system. This corresponds to the number of values in the “coordinate” sequence.

Like GM_Object, DirectPositions are associated with an SC_CRS through the “coordinateReferenceSystem” association. This association is typically not used since DirectPositions, as data types, will usually be included in larger objects (such as GM_Objects) that have their own references to SC_CRS. When this association is left NULL, the coordinate reference system of the DirectPosition instance will take on the value of the containing object’s SC_CRS.

One limitation of the DirectPosition class is that it does not support complex numbers. However, since DirectPosition does have a “coordinateReferencesystem” association with SC_CRS, it should be possible to model complex numbers in the CRS definition as two orthogonal axis.

5.2.4.3. Features in 3D

Non-point Features which are not bound to a planetary surface have special requirements. These Features are capable of movement in three dimensions. Additionally, these Features have non-trivial three-dimensional shapes which may change over time. Therefore, the movement of the Feature and the shape of the Feature are two separate properties.

A measurement of movement would capture changes in location and orientation. Movement is measured from the perspective of an external observer. Therefore, movement should be specified using a coordinate reference system which is external to the Feature.

The shape of a Feature is independent of its location. A rigid body has the same shape regardless of where it is or who is observing it. The object's geometry should be self-contained. This requires use of an internal coordinate reference system.

This leads to the following two postulates.

Postulate 1: The Locational Attribute of a 3D Feature is a GM_Point which locates the origin of the local CRS within an external CRS.

Postulate 2: The Spatial Attribute of a 3D Feature is one or more GM_Objects which define the shape of the Feature in the local CRS.

5.2.4.4. 3D Geometries

ISO 19107 makes a distinction between a geometric object and the surface which contains that object. One advantage of this approach is that there can be multiple surfaces associated with one object. For example, an island located in a lake would be represented by an interior surface (the island) of a polygon (the lake) bounded by the exterior surface (the shoreline).

ISO 19107 uses the GM_Object class to define an object and the GM_Boundary class to define a containing surface. Both GM_Object and GM_Boundary are defined as root level geometry classes. The association between GM_Object and GM_Boundary is achieved through the "boundary()" operation on the GM_Object class. This operation is inherited by all subclasses of GM_Object.

In the case of a 3D Feature, GM_Solid is the subclass of GM_Object while GM_SolidBoundary is the subclass of GM_Boundary. GM_Solid describes the volume while GM_SolidBoundary describes the shape.

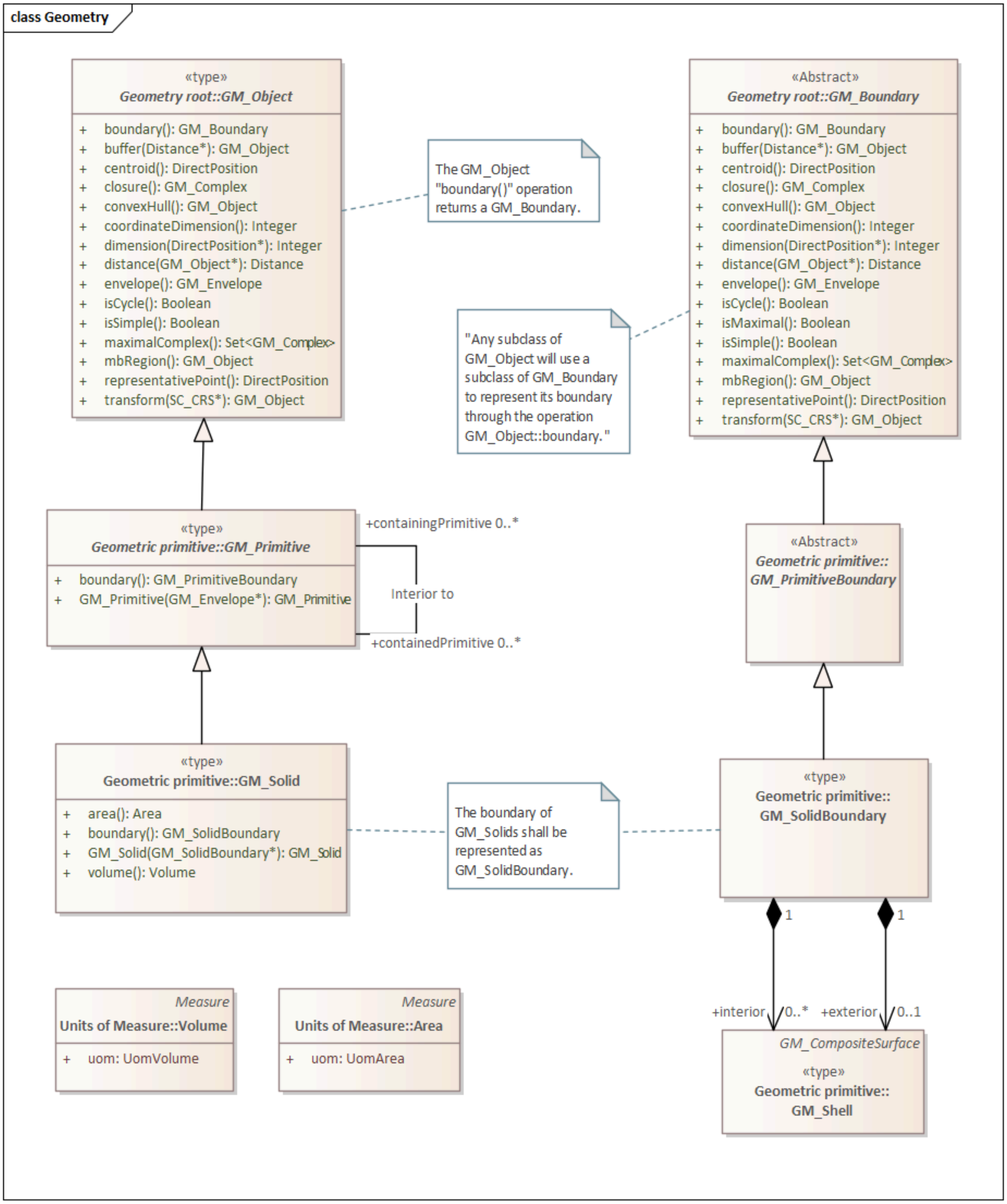

Figure 8 — 3D Geometry UML Model

5.2.4.5. Volumes

GM_Object is subclassed into GM_Primitive and then into GM_Solid. The “volume()” operation on GM_Solid returns the volume (defined in ISO 19103) of space contained within that GM_Solid. Thus, ISO 19107 supports the concept of a 3D volume.

Real 3D objects are often not solid. So the 3D model must also support voids, or even entire 3D Features within their interior. GM_Primitive addresses this need through the “interior to” association. The two roles on this association are the containingPrimitive (the GM_Primitive which contains another GM_Primitive) and the containedPrimitive (the GM_Primitive which is contained). This association has proven its worth in 2D space so there is little doubt that it will be just as effective in 3D.

5.2.4.6. Shapes

A 3D volume is delineated by a bounding surface. GM_Boundary is the root class for boundaries. The subclass GM_PrimitiveBoundary provides the boundary for GM_Primitives. The GM_PrimitiveBoundary subclass GM_SolidBoundary is defined as the boundary for a GM_Solid.

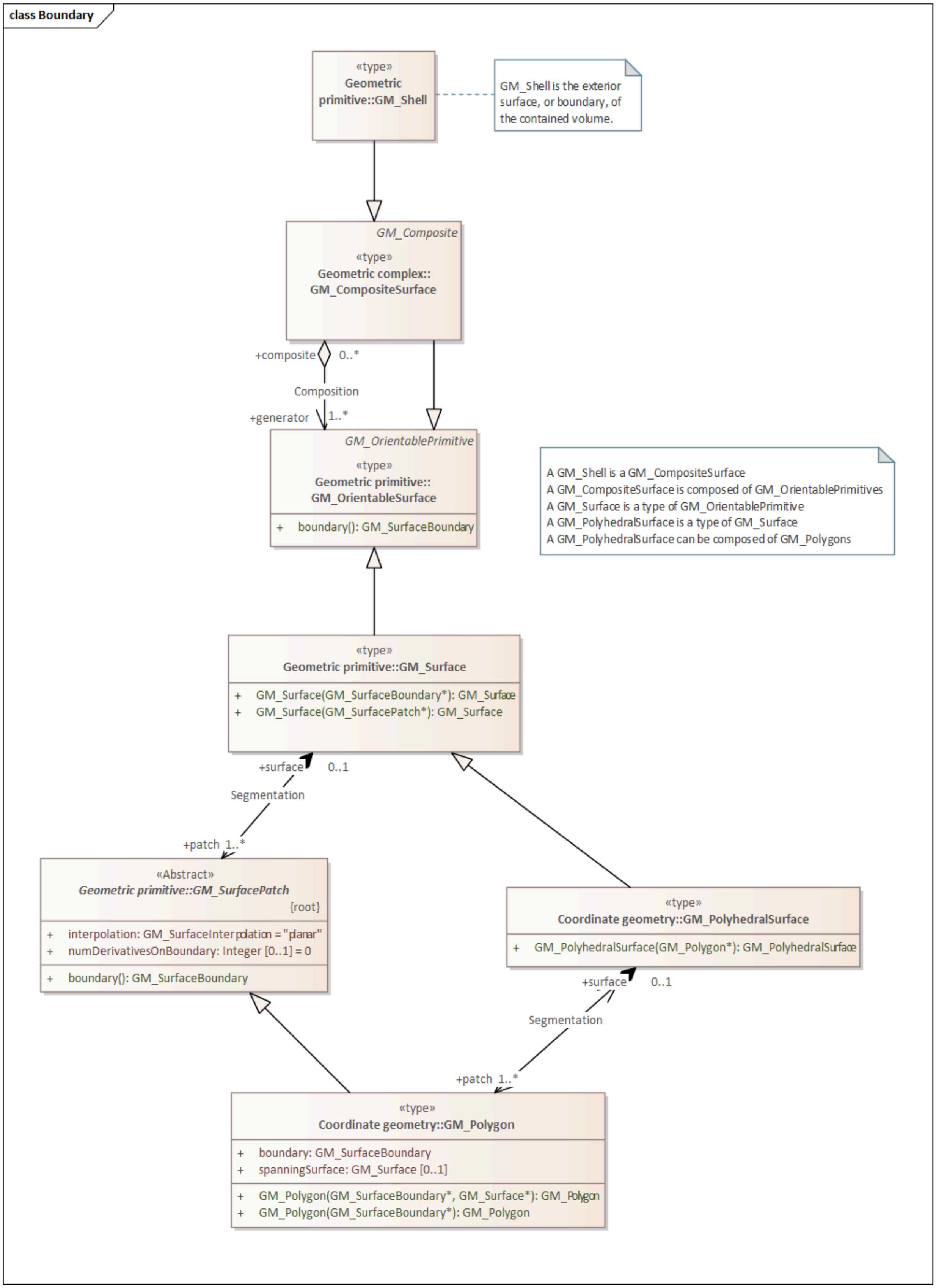

Figure 9 — 3D Boundaries UML Model

ISO 19107 goes even farther. A GM_SolidBoundary is composed of both interior and exterior boundaries. These boundaries are defined by the GM_Shell class. Therefore, the following can be observed.

- A GM_Shell is a GM_CompositeSurface
- A GM_CompositeSurface is composed of GM_OrientablePrimitives
- A GM_Surface is a type of GM_OrientablePrimitive
- A GM_PolyhedralSurface is a type of GM_Surface
- A GM_PolyhedralSurface can be composed of GM_Polygons

A GM_PolyhedralSurface which is composed of GM_Polygons is an example of Boundary Representation (B-Rep) of a surface. This approach is fundamental to rendering 3D computer graphics. (ref Adam Powers 1981)

5.2.4.6.1. Closure Surfaces

Some structures, such as a tunnel or overpass, pose difficulties for this geometry model. The boundary surface can be constructed so that it continues into the interior of the structure. That would make the interior of a tunnel external to the tunnel object. This is not always a desireable result. CityGML 3.0 addresses this issue thought the concept of a “Closure Surface”.

A Closure Surface is a surface which is a logical part of the object but does not correspond to a physical part of the object. For example, the entrance to a tunnel can have a closure surface. This surface allows the tunnel to be treated as a three-dimension solid, even though there is a hole in the bounding surface.

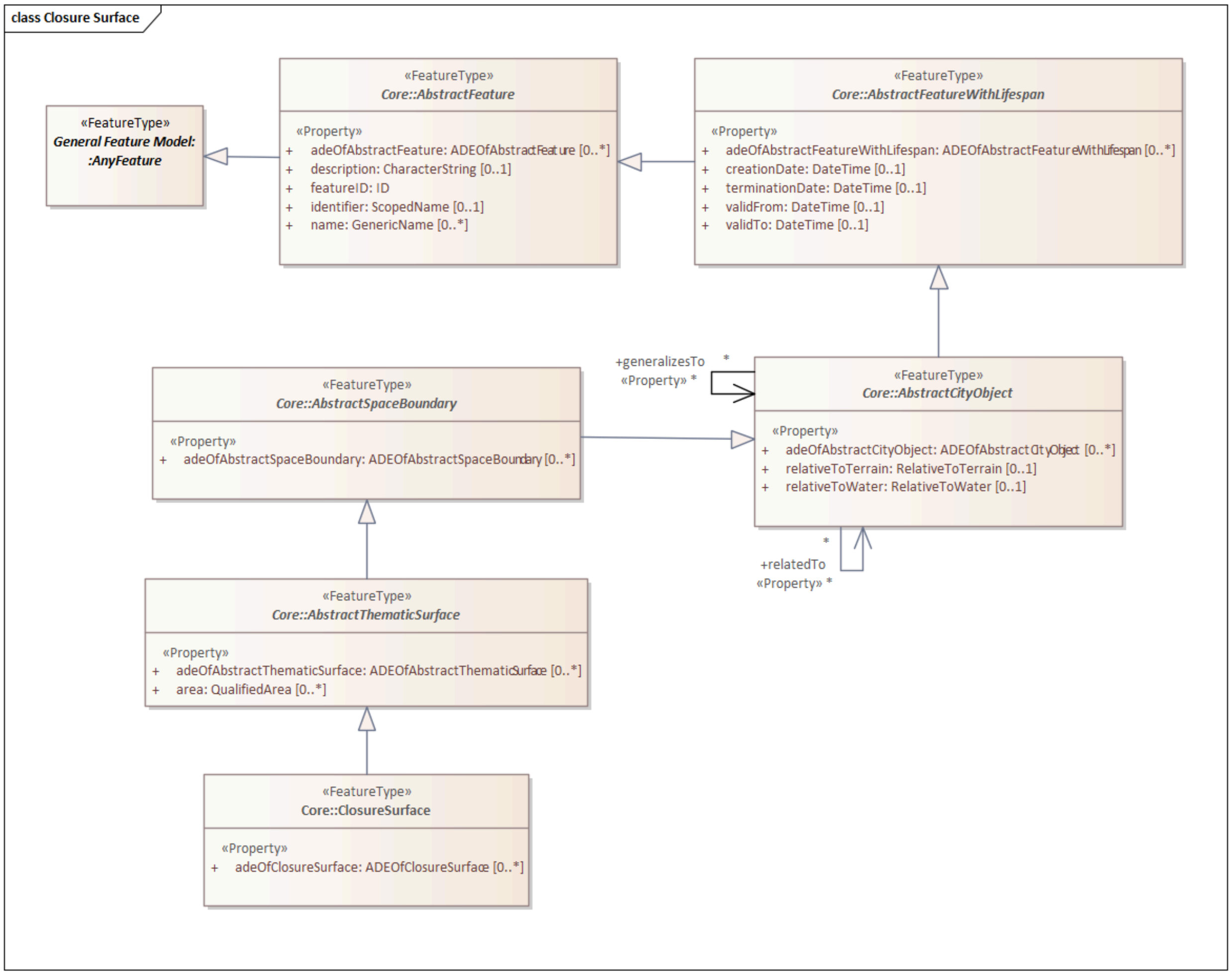

Figure 10 — Closure Surface UML Model

As implemented in CityGML 3.0, the ClosureSurface class an impressive history. This concept might need to be generalized for use outside of CityGML. However, the capabilities provided by the ancestor classes do provide value and may be worth incorporating into a general 3D model.

5.2.4.6.2. B-Rep

The polyhedral surfaces which bound volumetric shapes are similar to the Boundary Representation (B-Rep) approach used in CAD and computer graphics. B-Rep defines a 3-dimensional surface which serves as the interface between the interior of the volumetric shape and the exterior. This surface is usually defined by a collection of shape elements (polygons) which together form a closed surface.

https://en.wikipedia.org/wiki/Boundary_representation

5.2.4.6.3. Point Clouds

Boundary surfaces can also be defined using 3D point clouds. This allows the spatial representation of a bounding surface by a set of points located on that surface. In this way, the geometry of a Feature could, for instance, be modeled directly from the result of a mobile laser scanning campaign.

5.2.5. Moving Features

The ISO Standard for Moving Features is ISO 19141:2008 Geographic information — Schema for moving features. This Standard extends the geometry model from ISO 19107 and, by association, the Feature Model from ISO 19109.

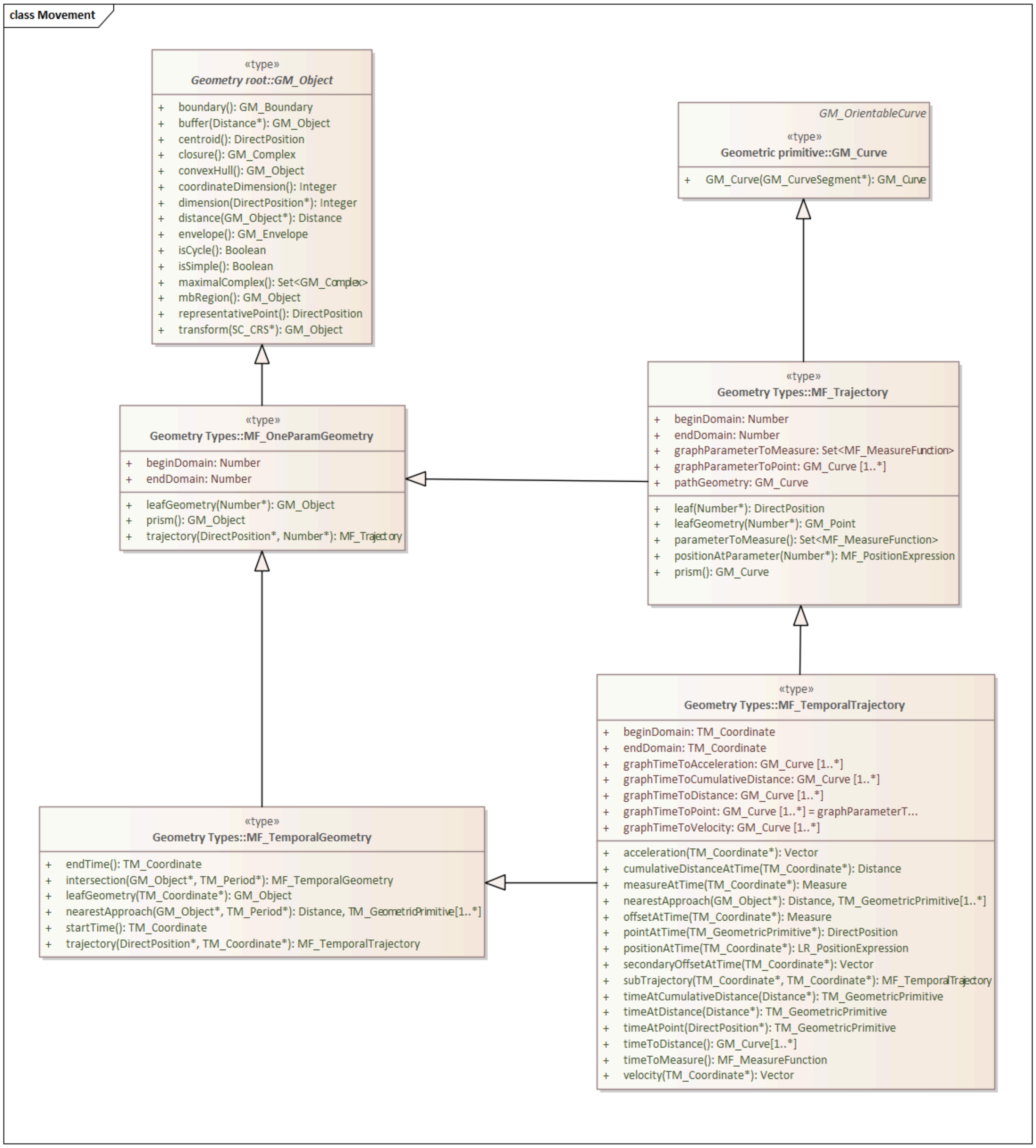

Figure 11 — Moving Features

A high-level view of ISO 19141 is provided in Figure 11. The classes identified in this figure are described below. But first, a discussion of coordinate reference systems (CRS) is in-order.

5.2.5.1. Coordinate systems for Moving Features

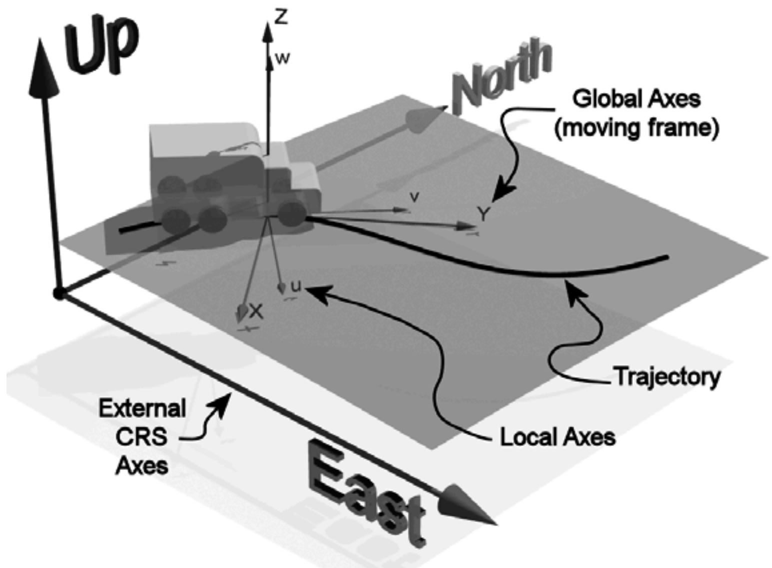

Figure 12 — External, Global, and Local CRS

Moving Features deal with three spatial coordinate systems as well at one temporal coordinate system. The spatial coordinate systems are referred to as the External, Global, and Local CRS (see Figure 12).

When dealing with Moving Features, converting coordinates between the three spatial CRS is frequently required. The GM_Object class provides the transform() operation which can be used for this purpose.

5.2.5.1.1. External CRS

The External coordinate system is the coordinate system within which the Moving Feature exists. Typically, this is an Earth-centric geographic CRS such as WGS 84. In the ISO 19111 model this would be a geodetic coordinate reference system.

5.2.5.1.2. Local CRS

The local coordinate system is internal to the Feature. This is usually a cartesian coordinate system with the origin at a prominent point in the Feature such as the center of mass. In the ISO 19111 model this would be an engineering coordinate reference system.

For rigid bodies, the local coordinate system is fixed over time. It does not change regardless of any motion by the associated Feature.

5.2.5.1.3. Global CRS

The Global coordinate system provides a transition between the External CRS and Local CRS. Global CRS is a moving CRS which follows the trajectory of the Moving Feature. As such, it provides the External CRS a time variant local reference system, while providing a time invariant context for the Local CRS.

The origin of the Global CRS is the location of the Moving Feature on the trajectory curve at a specific time. From the perspective of the External CRS, the origin translates and rotates as a function of time. From the perspective of the Internal CRS, however, the origin of the Global CRS is static.

The axis of the Global CRS can be defined using two techniques.

The first approach treats the Moving Feature as a black box viewed by an external observer. From this perspective, the Global CRS is defined in terms of the trajectory alone. No knowledge of the properties or even the shape of the Moving Feature are required. This CRS starts by defining x and y as two orthogonal axis which define a plane tangential to the Trajectory curve at the origin. Positive x is in the direction of motion. The positive y axis is perpendicular to the x axis and forms either a right or left handed CRS. The z axis is perpendicular to the tangent plane. Positive z can be either up or down. The result is a cartesian reference system which is always tangential to the trajectory of the Moving Feature.

The second approach defines the Global CRS in terms of motion properties of the Moving Feature. The x axis, for example, could be defined as the heading of the Moving Feature. This is the direction the Feature is pointing, but not necessarily the direction it is moving. The y and z axis can be defined using similar properties (for example; pitch, yaw, and roll).

In general, the black box approach is appropriate for ballistic Moving Features such as in cases where the Feature is not anticipated to take any actions which would modify the trajectory. The motion properties approach is appropriate for navigated Moving Features such as in cases where a Feature is expected to take actions which would modify the trajectory.

5.2.5.2. Temporal Reference Systems

ISO 19108:2006 Geographic Schema — Temporal Schema is the ISO standard for Temporal Reference Systems, in particular, the TM_ReferenceSystem class.

TM_ReferenceSystem has two attributes: `domainOfValidity` and `name.` The `name` attribute is an identifier for this temporal reference system. The `domainOfValidity` specifies the spatial extent over which this TRS is applicable.

TM_ReferenceSystem is specialized through a number of subclasses. The two most relevant to this paper are TM_CoordinateSystem and TM_Clock.

TM_CoordinateSystem is “A system for measuring time on a continuous interval scale using a single standard time interval.” The standard time interval is provided through the `interval` attribute. In addition, the `origin` attribute provides a temporal “datum” from which time is measured. Since time is a one-dimensional quantity, the origin and interval are sufficient to define a basic Temporal Coordinate Reference System.

TM_Clock is “A system for measuring temporal position within a day.” It has an optional `dateBasis` association with a calendar (TM_Calendar).

This combination of classes supports high precision local-clock TRS as well as full date-time TRS.

5.2.5.3. Coordinate Representation

The coordinates used to define a 3D moving geometry (MG) face requirements specific to their use. These requirements are derived from two characteristics of moving geometries. Unlike static spatial geometries, time and location in moving geometries are tightly coupled. They must act as a single, four-dimension location. In addition, there will be a large number of coordinate measurements. This is a result of the need to accurately track movement over time.

1. An MF coordinate must represent a discrete location in space and time.
2. An MF coordinate must include values for all three spatial axis (X,Y,Z) as well as the temporal axis (t).
3. An MF coordinate must be concise.

Of the Moving Feature encoding standards, the JSON encoding comes closest to meeting these requirements. Its major shortfall is the need for conformance with GeoJSON. Since GeoJSON assumes a terrestrial spatial geometry, spatial and temporal coordinates must be encoded separately.

- A LinearTrajectory object SHALL be a GeoJSON Feature object that has two MANDATORY members of “geometry” and “properties.”

The spatial locations are captured using the GeoJSON “geometry” property. This property is restricted as follows.

- The value of the “geometry” member SHALL be a LineString Geometry object, having “type” = “LineString.”
- The number of elements in the array of the “coordinates” value in the Geometry object SHALL be more than two positions.

So the spatial geometry is a linestring of more than two points.

GeoJSON does not support temporal coordinates directly. So the “properties” property is adapted for this purpose. Since “properties” is not limited to temporal coordinates, these requirements are more complex.

- The value of the “properties” member SHALL be a GeoJSON object that has at least one member with the name “datetimes”.

- The value of the “datetimes” member is a JSON array.
- Each element in the “datetimes” array SHALL be an instant object.
- An instant object SHALL be a JSON string that represents a timestamp encoded in the IETF RFC 3339 format using `Z` or the numeric value of milliseconds since midnight (00:00 a.m.) on January 1, 1970,the beginning of the Unix epoch, in UTC.
- The members of the “datetimes” array SHALL be a monotonic increasing sequence.
- There SHALL be no instant object that has the same value as any other element.

The consequence of these requirements is that the “properties” property can carry the temporal equivalent to a line string. There is one final requirement.

- The number of elements in both arrays of the “coordinates” value and the “datetimes” value SHALL be equal.

So there is a one-to-one correspondence between the temporal measurements in the “properties” property and the spatial measurements in the “geometry” property.

An example of this encoding is provided in Figure 13.

```
{ "type": "Feature", "id": "A", "geometry": { "type":
"LineString", "coordinates": [[11.0,2.0,50.0], [12.0,3.0,52.0],
[10.0,3.0,56.0]] }, "properties": { "datetimes":
["2012-01-17T12:33:51Z", "2012-01-17T12:33:56Z",
"2012-01-17T12:34:00Z"], "state": ["walking", "walking"],
"typecode": [1, 2] } },
```

Figure 13 — Example Moving Feature Encoding

5.2.5.4. Time-Variant Geometry

The main capability that the Moving Features Standard introduces is the concept of geometries which vary with time. ISO 19141 achieves this capability through the addition of the MF_OneParameterGeometry and MF_TemporalGeometry classes to the geometry defined in ISO 19107.

5.2.5.4.1. MF_OneParameterGeometry

The definition of time-variant geometries begins with the class MF_OneParameterGeometry. MF_OneParameterGeometry is a subclass of GM_Object. So moving features have the 3D geometric properties of any other GM_Object. The difference is that this geometry can change as a function of a parameter.

A one parameter set of geometries is defined as follows.

“A function *f* from an interval $t \in [a, b]$ such that *f(t)* is a geometry and for each point $P \in f(a)$ there is a one parameter set of points (called the trajectory of P) $P(t) : [a, b] \rightarrow P(t)$ such that $P(t) \in f(t)$. A leaf of a one parameter set of geometries is the geometry *f(t)* at a particular value of the parameter.”

A one parameter geometry instance includes a “leafgeometry()” operation. This operation takes the parameter (*t*) as input and returns the leaf P(*t*) for that parameter as a GM_Object.

5.2.5.4.2. MF_TemporalGeometry

An MF_TemporalGeometry is a MF_OneParameterGeometry where the parameter is Time expressed as a TM_Coordinate. TM_Coordinate is specified in ISO 19108. It expresses time as a multiple of a single unit of measure such as year, day, or second. The “leafgeometry()” operation of an instance of MF_TemporalGeometry would take a TM_Coordinate in as input and return a GM_Object instance representing the geometry of the Feature at the specified point in time.

5.2.5.5. Temporal Properties

The OGC Moving Features JSON encoding standard introduces the concept of temporal properties.

“A TemporalProperties object is a JSON array of ParametricValues objects that groups a collection of dynamic non-spatial attributes and its parametric values with time.”

Logically TemporalProperties should be a subclass of MF_OneParamProperties. Since Geometry is a property, then MF_TemporalGeometry should be a subclass of TemporalProperties, which results in the following UML.

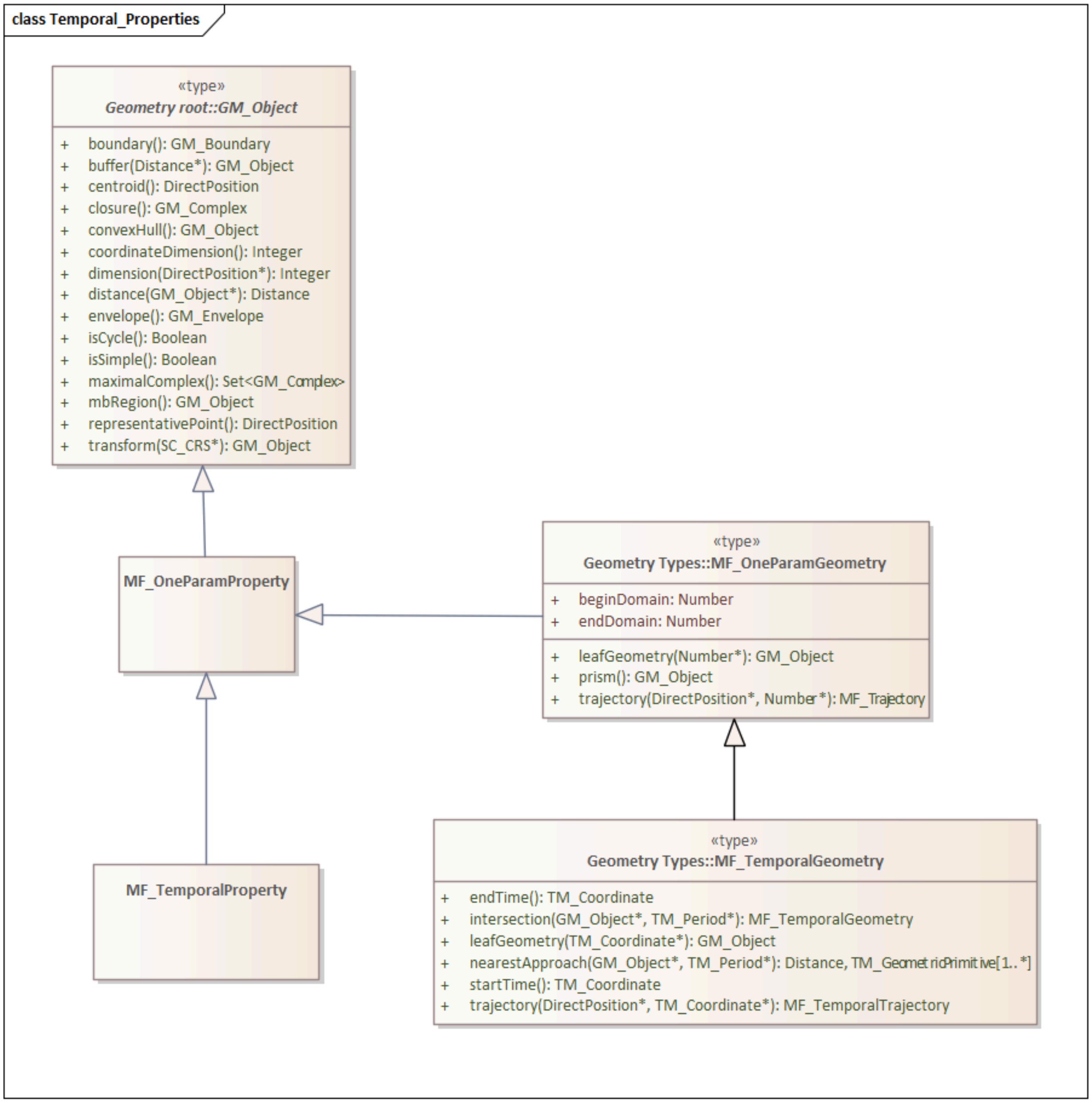

Figure 14 — Temporal Properties

Temporal properties are particularly useful for capturing state change. For example, the fuel load of an aircraft will change over time. The leafproperty() operation on a temporal fuel_load object would return the amount of fuel onboard at the specified time.

5.2.5.6. Location

ISO 19141 represents the location of a Moving Feature using two classes: MF_Trajectory and MF_TemporalTrajectory.

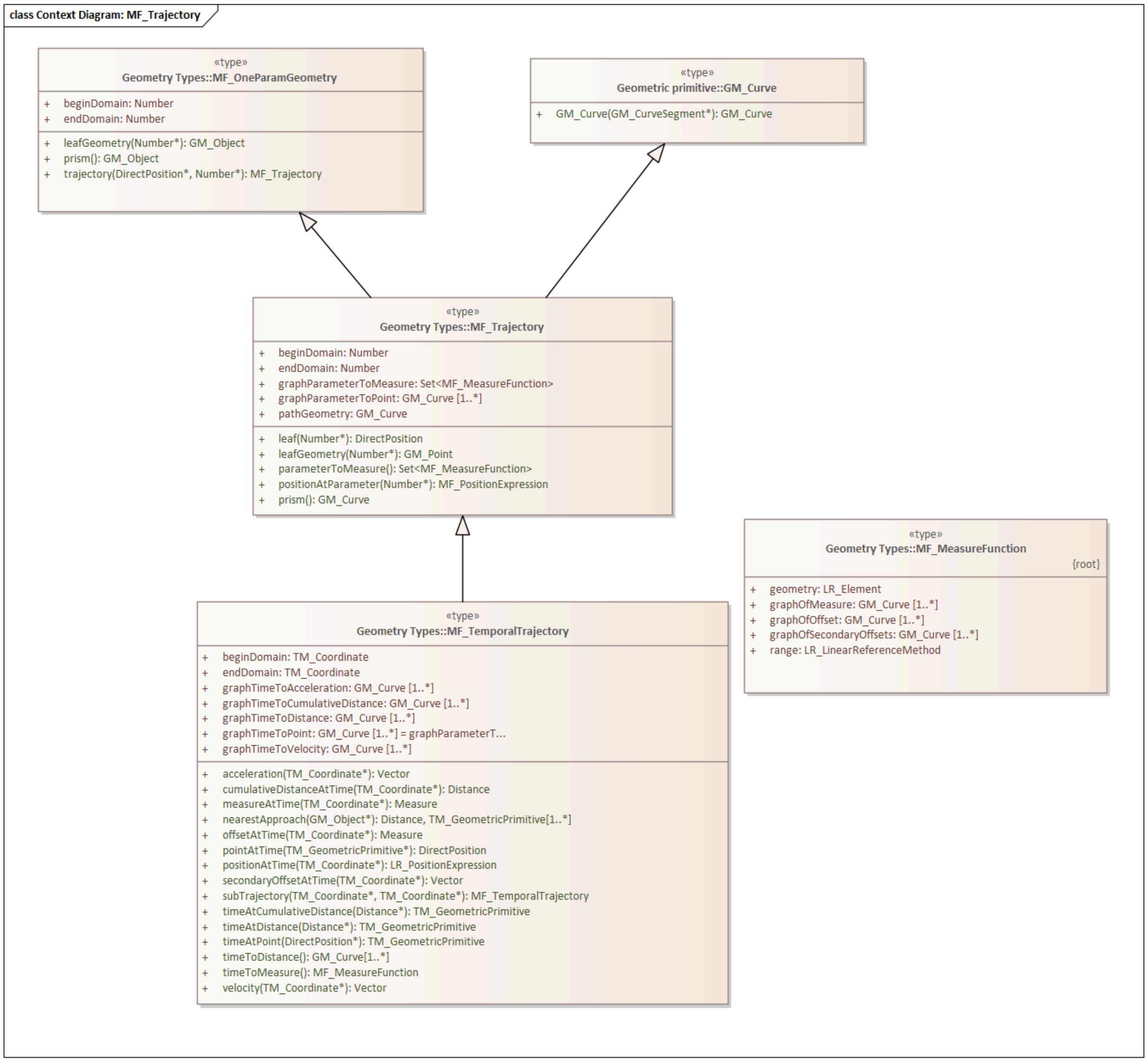

Figure 15 — Trajectory

A MF_Trajectory is a curve (GM_Curve) representing every position that the Feature has occupied during it's journey and does not necessarily represent the time when each location was reached.

MF_TemporalTrajectory makes the MF_Trajectory a MF_TemporalGeometry which represents location along the trajectory as a function of time. Therefore, each location is fully defined in both space and time.

A Temporal Trajectory has two operations of particular interest; leaf() and leafgeometry(). The input parameter for these operations is always time (TM_Coordinate). The leaf() operation returns the spatial location (Direct_Position) that the Moving Feature passes at the time (TM_Coordinate) specified by the input parameter which is a point on the trajectory GM_Curve geometry. The leaf() operation also serves as the origin of the Global CRS at that location on the trajectory.

The LeafGeometry() operation returns the spatial geometry (GM_Point) that this Moving Feature possesses at the time (TM_Coordinate) specified by the input parameter is the shape of the Moving Feature expressed in the Local CRS. Since Trajectories only convey location, only GM_Point geometries are supported.

5.2.5.7. Orientation

5.2.5.7.1. MF_PrismGeometry

If an application focuses on only the linear movement (i.e., the spatiotemporal line string) of moving points based on World Geodetic System 1984, with longitude and latitude units of decimal degrees, and the ISO 8601 standard for representation of dates and times using the Gregorian calendar, the application can share the trajectory data by using **only** IETF GeoJSON, called **MF-JSON Trajectory**. For other cases, **MF-JSON Prism** can be used for expressing more complex movements of moving features. **MF-JSON Prism** is a GeoJSON-like format reserving new members of JSON objects ("temporalGeometry," "temporalProperties," "crs," "trs," "time," and others) as "foreign members" to represent spatiotemporal geometries, variations of measure, coordinate reference systems, and the particular period of moving features in a JSON document.

A trajectory provides the location of a Moving Feature as a function of time. Prism Geometry represents the full geometry (location, orientation, and shape) of the Feature as a function of time.

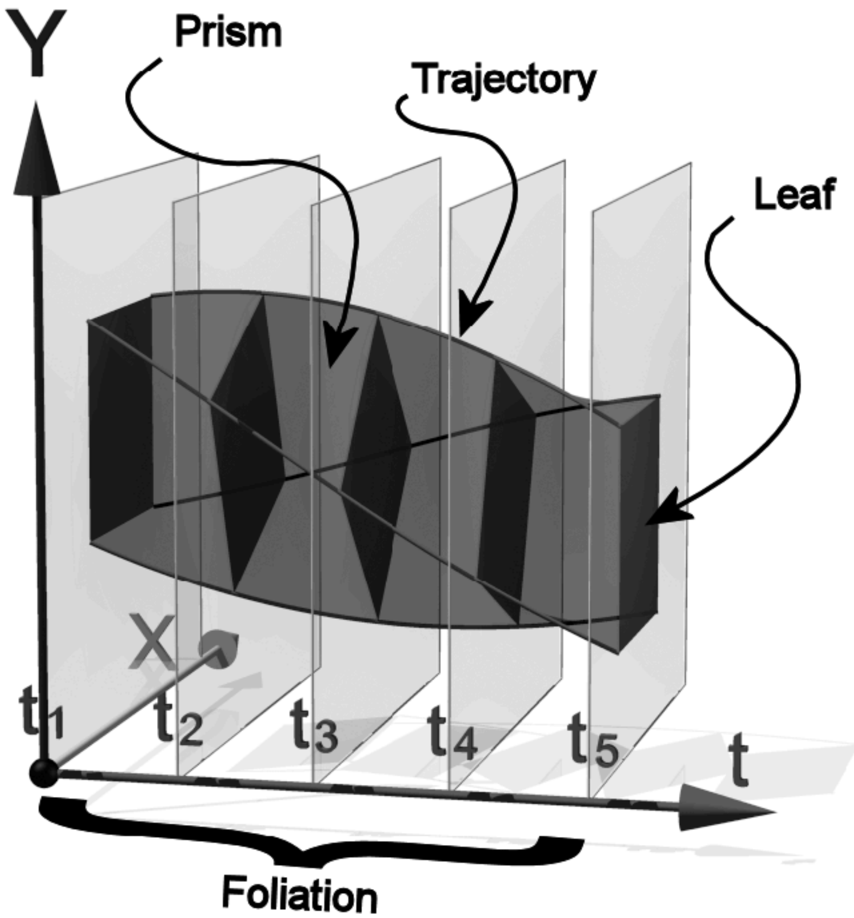

Figure 16 — Foliation

The key concepts in the Prism model are as follows.

Leaf: A leaf is the geometry of the Moving Feature at time (tn).

Foliation: A collection of leaves where there is a complete and separate representation of the geometry of the Feature for each specific time (tn).

Trajectory: A curve that represents the path of a point in the geometry of the Moving Feature as it moves with respect to time (t).

Prism: The union of the geometries (or the union of the trajectories) in a foliation.

Like a Temporal Trajectory, a Prism is a subclass of MF_TemporalGeometry.

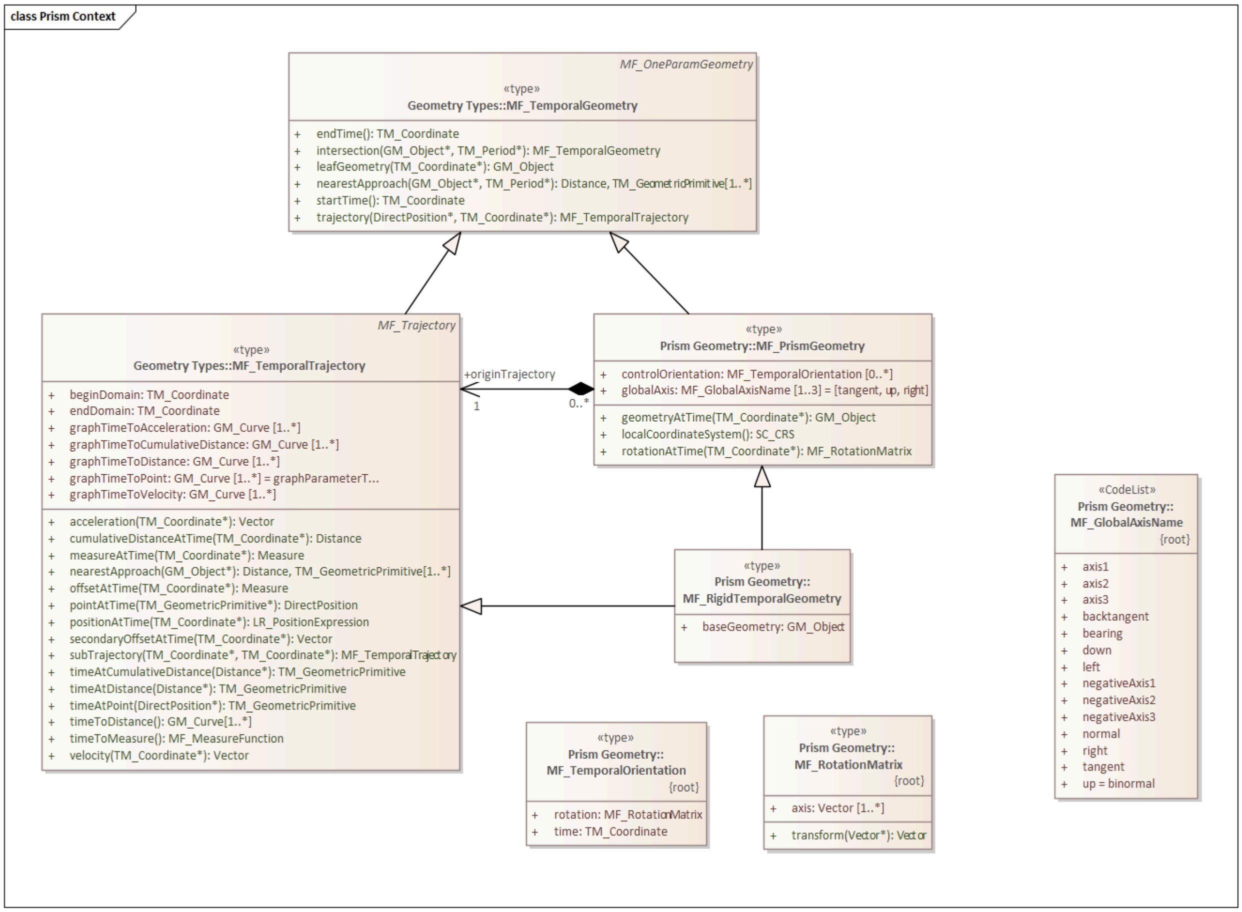

Figure 17 — Prism Context

A MF_PrismGeometry class has the following characteristics.

The association role “originTrajectory” associates a Temporal Trajectory with a Prism geometry. For any TM_Position the following hold true.

1. The associated Temporal Trajectory provides the location of the Moving Feature in the Global CRS.
2. This location serves as the origin of the Local CRS.
3. The prism geometry is defined in that specific Local CRS.

The localCoordinateSystem() operation returns a SC_CRS for the design coordinate reference system in which the moving feature’s shape is defined. This is usually the same as the local coordinate system.

The rotationAtTime() operation accepts a time in the domain of the prism geometry and returns the rotation matrix that embeds the local geometry into geographic space at a given time (TM_Coordinate). The vectors of the rotation matrix allow the feature to be aligned and scaled as appropriate to the vectors of the global coordinate reference system.

This one association and two operations provide the location, orientation, axis definition, and units of measure needed to define the local CRS and to transform geometries between the Local and Global CRSs.

Finally, the geometryAtTime() operation accepts a time in the domain of the prism geometry and returns the geometry of the moving feature, as it is at a given time in the global coordinate reference system. The return type is a GM_Object so this operation is not limited to points. It is fully capable of representing a 3D surface and volume.

In short, a MF_PrismGeometry provides the shape, location, and orientation of a Moving Feature as a function of time (tn).

5.2.5.8. Non-rigid Bodies

ISO 19141 only addresses rigid bodies. The shape returned by a geometryAtTime() operation will always be the same. However, it leaves open the opportunity to extend the Moving Feature model to support plastic (non-rigid) objects.

The most obvious approach is to allow the geometry returned by the geometryAtTime() operation to change as a function of time. This doesn't require a change to the model, but may require some changes to the standard.

The geometry itself could include MF_TemporalGeometry elements. These elements would each have their own lifespan and a history of their movement, in respect to the local CRS, over time.

5.3. OGC GeoPose Draft Standard

Given a suite of standards that support defining time-variant geometric elements, the next step is to take a collection of those elements and assemble them into a complex object. GeoPose is a proposed OGC standard that addresses this requirement. GeoPose deals with the location and orientation of real or virtual geometric objects (Poses) and the aggregation of Poses into more complex structures.
All UML diagrams about GeoPose concepts are provided in the annex, starting with the core GeoPose model. The key element in the core GeoPose model is the `FrameTransform` class. This class expresses a transform between a pair of Reference Frames, Outer and Inner, both anchored to the Earth's surface or to other bodies.
The Frame Transform is a representation of the transformation taking an Outer Frame coordinate system to an Inner Frame coordinate system. GeoPose v 1.0 supports transformations involving translation and rotation. The intention is to match the usual concept of a pose as a position and orientation. Outer and Inner Frames are subclasses of the the Frame Class. The Frame class provides a standard means of describing transforms for several common coordinate reference systems. Subclasses of the Frame class also include a model for generic reference frames.

The GeoPose time model is rather simple. GeoPose does not use the calendar and restricts time positions to milliseconds of UNIX Time. Unix time is a date and time representation widely used in computing. It measures time by the number of milliseconds that have elapsed since 00:00:00 UTC on 1 January 1970.

[**SOURCE:** OpenGroup]

In GeoPose 1.0, that outer frame is fixed to a three-dimensional “World Geodetic System 1984” CRS with latitude, longitude, and ellipsoidal height axes (EPSG:4979).
The sequence logical model defines methods for packaging GeoPose transformations. It addresses the need to integrate multiple GeoPoses which share the same Outer Frame and possess a time-dependent changing Inner Frame.

The GeoPose Standard provides three models for organizing a collection of Inner Frames.

- Stream: The Inner Frame definition (Frame) and an associated time stamp are delivered sequentially.
- Regular Series: The Inner Frame definitions are delivered as a sequence, separated by a fixed time interval.
- Irregular Series: The Inner Frame definitions and associated time stamps are delivered as a collection. There is no explicit spatial or temporal order to the frames.

5.4. CCSDS Conventions

The most important CCSDS conventions regarding coordinate reference systems are in the informational report on **Navigation Data — Definitions and Conventions**. This document was first published in 2001 and last revised in 2019 (as for December 2022).

5.4.1. Coordinate System Definitions and Specifications

According to the CCSDS, in order to define a coordinate system unambiguously the following elements are needed.

- A **coordinate frame** which is defined as an associated set of mutually orthogonal Cartesian axes.
- A **frame origin** which is the common origin of the Cartesian axes (e.g., center of mass of Earth, satellite etc.).
- A **reference plane** which is the xy-plane in a coordinate frame and therefore defines the direction of the z-axis such as the Earth equator or the elliptical plane.
- A **reference direction** which defines the direction of the x-axis (e.g., vernal equinox).

The definition of a reference plane and direction can be done in different ways. Some realizations would be as follows.

- Pointing to a fixed direction in inertial space (e.g., toward a quasar)
- Parallel to the distance vector between one object and another
- Parallel to an object's velocity vector
- Pointing from the origin through the intersection of two defined planes
- Parallel to an object's spin axis
- Along a body axes
- Normal to an object's orbit

In current practice, most navigation messages between agencies use only a small subset of reference systems and frames. The most used are the International Celestial Reference System (ICRS) and the International Terrestrial Reference System (ITRS) defined by the IERS. A detailed discussion follows later in this report. In short, the ICRS and ITRS could be summarized as follows.

ICRS Inertial barycentric reference system whose axes are defined by the measured positions of extragalactic sources (mainly quasars).

ITRS Terrestrial Earth-fixed reference system for measurements on locations relatively near the Earth's surface.

Since many coordinate systems use some sort of equator as the reference plane, motions of this plane must be compensated for. The effects can be separated into long-term (precession)and short-term effects (nutation). Due to those effects, the CCSDS distinguishes between *True of Date* (TOD) and *Mean of Date* (MOD). The mean equator and vernal equinox of a given date define a MOD coordinate system, which includes long-term, but not short-term effects. A given date's true equator and equinox, which can be obtained by applying short-term effects to the mean values, define a TOD coordinate system. A detailed description of long- and short-term effects is done in Clause 6. The *Greenwich True of Date Coordinate System* (GTOD) is the last important coordinate reference system on the list. In contrast to different realizations of the ITRS (the so-called frames, e.g., ITRF2000), the GTOD corresponds to the respective true point in time. GTOD is defined as follows.

- Frame origin: Center of the Earth
- Reference plane: The Earth's true-of-date Equator (therefore the z-axis is directed along the Earth's true-of-date rotational axis and is positive north)
- Reference direction: The positive x-axis is directed toward the prime meridian
- The y-axis completes a right-handed system

Important reference frames defined using the orbital position and velocity at a given time are used in aerospace for attitude estimation, attitude control, and orbital relative motion. All the local orbital frames described in this convention are rotating coordinate systems unless specified otherwise in the context of some specific data exchange between participants. These systems can be used to study the relative motion between spacecraft.

5.4.2. Time

Since the CCSDS sees the world from the aerospace point of view,in contrast to the ISO Standard, a main chapter deals with coordinate systems in reference to time and time systems. In detail, the exact definition and understanding of time systems is essential for the following.

- The modeling of satellite orbits and attitude
- Processing of navigation data
- Satellite ground operations

Furthermore, different time systems are represented in the convention. Since those are specified in detail in Clause 8, here a detailed representation is renounced. The time systems can be summarized as follows.

Terrestrial Time (TT)	Conceptually uniform time scale that would be measured by an ideal clock on the surface of a geoid.
Geocentric Time (GT)	Differs from Terrestrial Time by removing the effects of the Earth’s motion.
International Atomic Time (TAI)	Practical realization of a uniform time scale based on atomic clocks and agrees with TT (except for offset of 32.184s).
GPS Time	Differs from TAI in the chosen offset and the choice of atomic clocks used in its realization.
Greenwich Mean Sidereal Time (GMST)	Defined as the Greenwich hour angle of the mean vernal equinox of date.
Universal Time (UT)	Defined as solar time,
Universal Time Coordinated (UTC)	Atomic time, which is adapted to the universal time.
Barycentric Dynamical Time (TDB)	Independent variable of current barycentric solar system ephemerids.
Barycentric Coordinate Time (TCB)	Relativistic time coordinate of the 4-dimensional barycentric coordinate system.

5.5. NASA SPICE Tool

The last important aspect in this chapter is SPICE. In contrast to the other standards, specifications, and conventions, SPICE is a tool. SPICE is an ancillary information system that provides scientists and engineers the capability to include space geometry and event data into mission design, science observation planning, and science data analysis software. SPICE was originally developed at NASA’s Navigation and Ancillary Information Facility (NAIF), covered previously. It is used in a wide field of planetary missions (from all space agencies) and can be seen as the standard toolkit for this task. For the purpose of interplanetary coordinate calculations, multiple frames, times, and transformations between those must be done. A web-interface for SPICE-Toolkit is provided via WebGeocalc.

One of the main SPICE concepts are **kernels**. Kernels are the SPICE files, containing all the information needed for computation. These kernels provide information such as the following.

- Spacecraft trajectory and orientation
- Target body ephemeris, size, and shape
- Instrument field-of-view size, shape, and orientation
- Specifications for reference frames
- Tabulations of time system conversion coefficients

5.5.1. Time Systems

Every event that can be calculated in SPICE is associated to an epoch. An epoch is an instant in time. Clocks are used for the realization of this. Clocks count epochs specified by events such as: “regular” oscillations of a pendulum, quartz crystal, or electromagnetic radiation from a specified source, measured from an agreed upon reference epoch. A time system is an agreed upon standard for “naming” epochs, measuring time, and synchronizing clocks. SPICE divides all time systems into three categories.

- Ephemeris Time (ET) or (the newer name) Barycentric Dynamical Time (TDB)
- Universal Time Coordinated (UTC) which corresponds to the standard time when using calendars
- Spacecraft Clocks

The transformations between those time systems are implemented in the SPICE-toolkit.

5.5.2. Reference Frames and Coordinate Systems

The next important calculation step is the definition of the reference frame in which the calculations should take place. In SPICE, a reference frame is an ordered set of three mutually orthogonal (possibly time dependent) unit-length direction vectors, coupled with a location called the reference frame’s *center* or *origin*. All reference frames in SPICE are right-handed and a reference frame’s center must be a SPICE ephemeris object whose location is coincident with the origin of the frame. Reference frames can be divided into two main columns.

5.5.2.1. Inertial Reference Frames

A detailed discussion of the term *inertial* follows later in the ER. In a SPICE-sense the most important facts are that an inertial reference frame is **non-rotating** (with respect to fixed stars) and has a **non-accelerating origin** (velocity is typically non-zero, acceleration is negligible). A distinctive aspect of SPICE is the assumption that the center of any inertial frame is always defined by the solar system barycenter.
The most known example is the ICRF or J2000 reference frame (B1950). Strictly speaking, those two reference frames are not identical, but since the error is smaller than 0.1 arc seconds it will be neglected.

5.5.2.2. Non-inertial Reference Frames

Non-inertial reference frames are basically everything except for inertial reference frames. The following are several sub-groups of non-inertial reference frames.

- Body fixed, which means they are tied to a named body and rotate with it (e.g., Sun, planet, satellite, comet, asteroid)
- Topocentric, which would be placed on or near the surface of an object (e.g., Earth)
- Spacecraft, which is associated with the main spacecraft structure
- Instrument, which is associated to an instrument. There are one or more frames usually associated with each instrument (e.g., spacecraft antenna, solar array)
- Dynamic, which is a special feature of SPICE using time-dependent orientation

5.5.2.3. Coordinate Systems

A coordinate system specifies the method of locating a point within a reference frame. This definition contrasts with definitions of the IERS and other conventions and therefore is only used to describe SPICE. The main types of coordinate systems are as follows.

- Planetocentric Coordinate System
- Planetodetic Coordinate System
- Planetographic Coordinate System

Since SPICE strongly suggests not using planetographic coordinate systems, they will not be discussed further. The difference between planetocentric and planetodetic coordinate systems are similar to the difference between geocentric and geodetic latitudes, which will be discussed in the Figure 23 section.

> ***CAUTION***
>
> *Dwarf planets are often treated differently. Dwarf planets include: Ceres, Eris, Haumea, Makemake, and Pluto. For more information see SPICE documentation.*

6

HIERARCHY OF GEODETIC COORDINATE SYSTEMS AND FRAMES

6 HIERARCHY OF GEODETIC COORDINATE SYSTEMS AND FRAMES

In this chapter several coordinate systems relevant in geodesy are listed and defined. In geodesy, the following three classes of reference system are identified.

- Space-fixed system (subscript i = inertial),
- Earth-fixed system (subscript e = Earth-fixed),
- Local system (subscript g = gravity).

Figure 18 gives an overview of how subdivisions are defined. The color scheme indicates which kind of coordinates are used, although they are primarily Cartesian after a transformation. The figure is divided vertically into true/apparent and defined systems.

A rotation between space-fixed and Earth-fixed systems can be carried out only based on the momentary — the true — rotation axis of the Earth. Since the rotation axis wanders, both in space-fixed and in the Earth-fixed system and is therefore time-dependent, coordinate sets within these systems would have to be defined either time-dependently or as valid only at one point in time. In practice, the latter is completely unsuitable. A point on the Earth's surface would be in constant motion. A separation between a relative movement of the points among themselves and the movement due to the Earth rotation would be substantially more difficult, if not impossible. Therefore, the movement of rotation axis are most precisely defined relative to these "fixed" systems.

The left column lists inertial or space-fixed systems. The conventional inertial reference system represents the best approximation to a space-fixed reference system. Strictly speaking, it must defined as a quasi-inertial system, since practically no system is without proper motion. The relation between space-fixed systems is described by the two effects of precession and nutation.

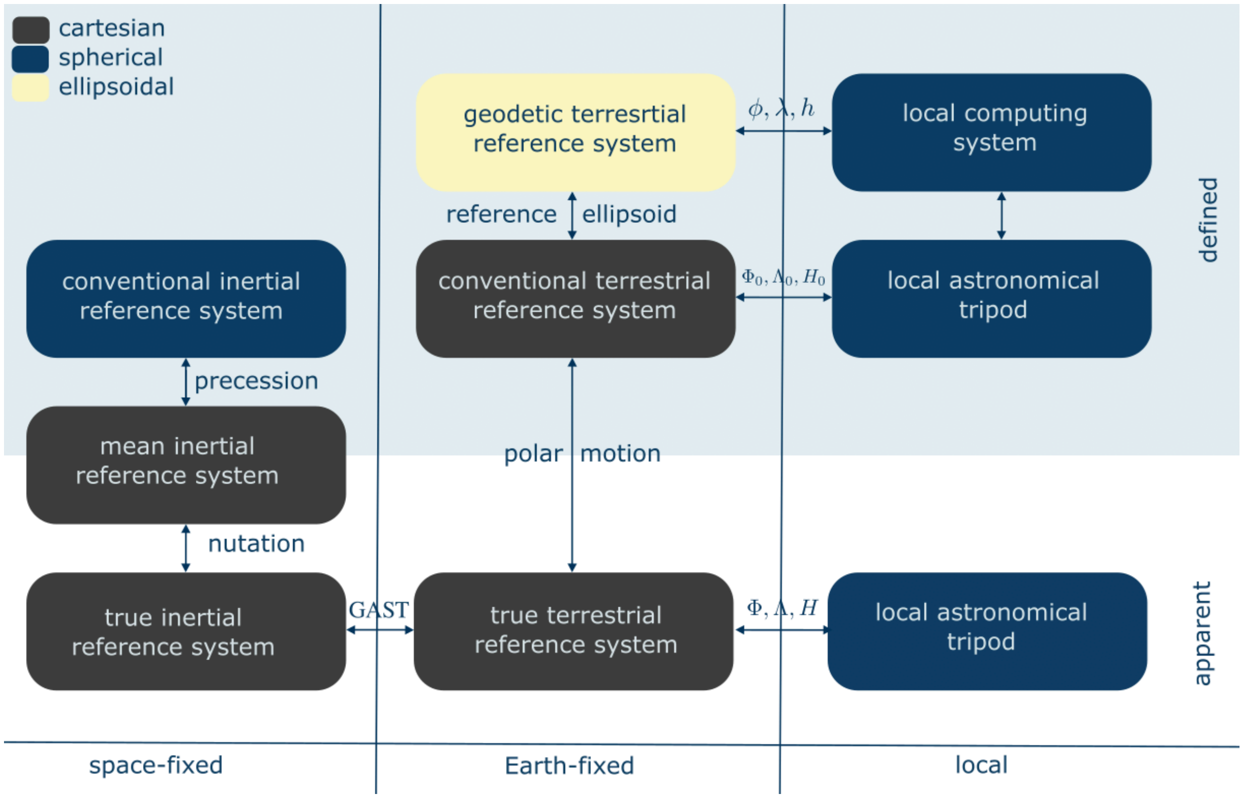

Figure 18 — Hierarchy of geodetic coordinate systems

Precession transitions a conventional inertial reference system defined at an epoch T_0 to the mean inertial system at epoch T. Given the nutation, the true inertial system at epoch T can be determined.

NOTE 1Precession

Summary Slow conical movement of the Earth's rotation axis around the NEP (North Ecliptical Pole) due to external torques.

Duration One full revolution of the NEP takes 25,765 years — a platonic year.

In general, precession is caused by constant external torques. In the case of the Earth, precession is caused by the Sun and Moon. The Sun's (or Moon's) gravitational pull on the nearest side of the Earth is stronger than the pull on the other side. At the same time the Earth is flattened. Therefore, if neither the Sun nor Moon are in the equatorial plane, a torque will be produced by the difference in gravitational pull on the equatorial bulges. Note that the Sun is only in the equatorial plane twice a year, namely during the equinoxes. The Moon goes through the equator plane twice per month. As a result of the constant (or mean) part of the lunar and solar torques, the angular momentum vector will describe a conical motion around the northern ecliptical pole (NEP). The northern celestial pole (NCP) slowly moves over an ecliptical latitude circle.

NOTE 2Nutation

Summary Nutation is a periodic (nodding) motion of the angular momentum vector in space on top of the secular precession.

Duration There are many sources of periodic torques, each with its own frequency.

1. **18.6 years**: The orbital plane of the Moon rotates once in this period under the influence of the Earth's flattening. The corresponding change in geometry also causes a change in the lunar gravitational torque of the same period. This effect is known as Bradley nutation.

2. **semiannual**: The Sun goes through the equatorial plane twice a year, during the equinoxes. At those times the solar torque is zero. Vice versa, during the two solstices, the torque is maximum. Thus, there will be a semi-annual nutation.

3. **annual**: The orbit of the Earth around the Sun is elliptical. The gravitational attraction of the Sun, and consequently the gravitational torque, will vary with an annual period.

4. **14-days**: The Moon passes the equator twice per lunar revolution, which happens approximately twice per month. This gives a nutation with a fortnightly period.

The subdivision of the Earth axis motion in the space-fixed system is artificial and finds its origin in physical causes. Today applications use a single rotation sequence. The term "true" is understood here in the sense of a system appearing momentarily to the observer. In other words, to the observer at the time T as momentarily true system.

The terrestrial systems are listed in the middle column. The transition from the true inertial to the true terrestrial system considers the rotation of the Earth around the actual rotation axis. There is a direct relation between the angle GAST and the time (see Clause 8). With the help of the polar motion the movement of the Earth axis in the Earth-fixed system is described and thus the transition of the true Earth-fixed system to a conventional Earth-fixed system can be accomplished. Since the use of longitude, latitude, and height above ellipsoid is more common with Earth-fixed coordinates, a reference ellipsoid is specified for the transition.

NOTE 3Polar motion

Summary Polar motion is free nutation, in other words a torque-free rotational motion that arises because the rotation axis is not perfectly aligned with the body axis that is defined by the moment of inertia. Viewed from inertial space, the Earth wobbles around the rotation axis.

Duration There are multiple periods of polar motion.

1. **435 days**: The so called Chandler wobble is due to the free oscillation of the body axis.

2. **annual**: Seasonal mass transports lead to an interference of annual and Chandler period.

3. **drift**: A polar drift of ca. 10 cm/year due to secular mass change, mostly caused by post-glacial rebound.

In the case of the Earth this results in a periodic circular motion of the Earth rotation axis around the third coordinate axis with respect to Earth-fixed coordinate system. In the inertial system, the body axis is rotating around the rotation axis, therefore the Earth wobbles.

The polar motion cannot be modeled, and its corrections must be determined daily. Often the true / instantaneous north pole is not used to transform between systems (see Clause 7). Instead, a reference north pole at a fixed point of time is used.

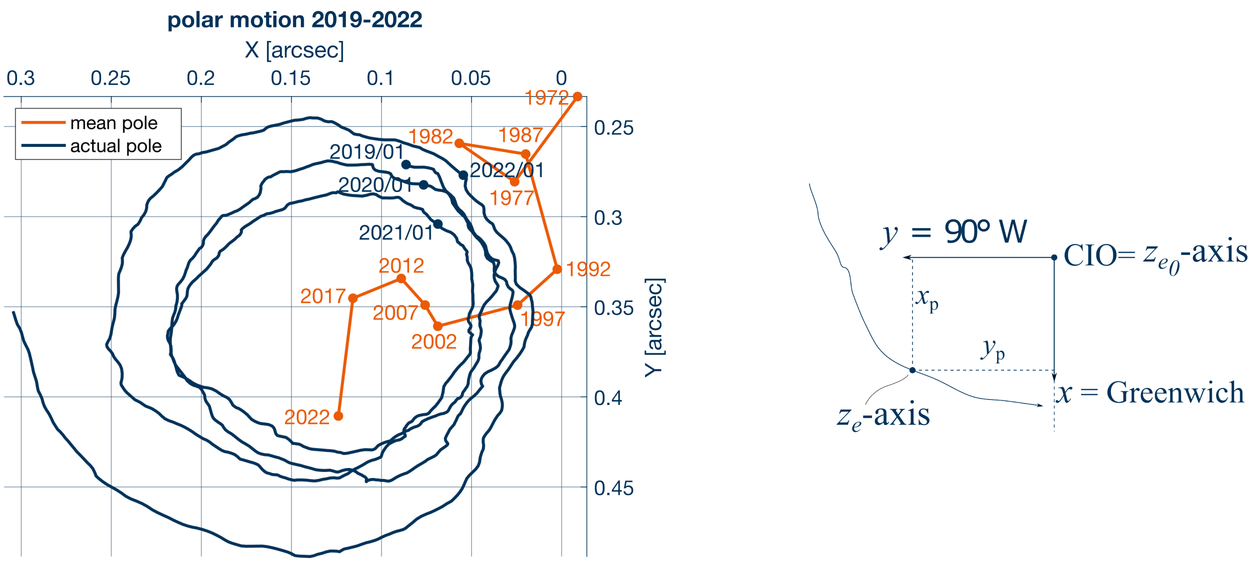

Figure 19 — Polar motion from 2019 to January, 2022 and mean pole up to the year 2022.

[**SOURCE:** IERS]

For the sake of completeness, the local systems are listed in the right column. The index g is used because instruments in a local horizontal system are aligned according to the plumb line, which is influenced by the gravity vector. The transformation from the Earth-fixed system is done either with the help of the ellipsoidal coordinates (ϕ, λ, h) or the astronomical latitude and longitude (Φ_0, Λ_0) and the orthometric height H_0 at the time T or T_0.

NOTE 4vernal equinox

The vernal equinox (also called vernal point) is the direction on the intersection between the ecliptic plane and the equatorial plane, where the Sun passes from southern to northern hemisphere. The vernal equinox is marked with the Aries symbol ♈#. The vernal equinox shows the position of the Sun on the celestial sphere as seen from Earth at the beginning of spring on the 20th / 21th of March. At the time of the determination of the zodiac signs about 2000 years ago, the Sun was in the zodiac sign of Aries at the beginning of spring. In the meantime, due to the precession of the Earth's axis, the vernal equinox has moved to the constellation of Pisces.

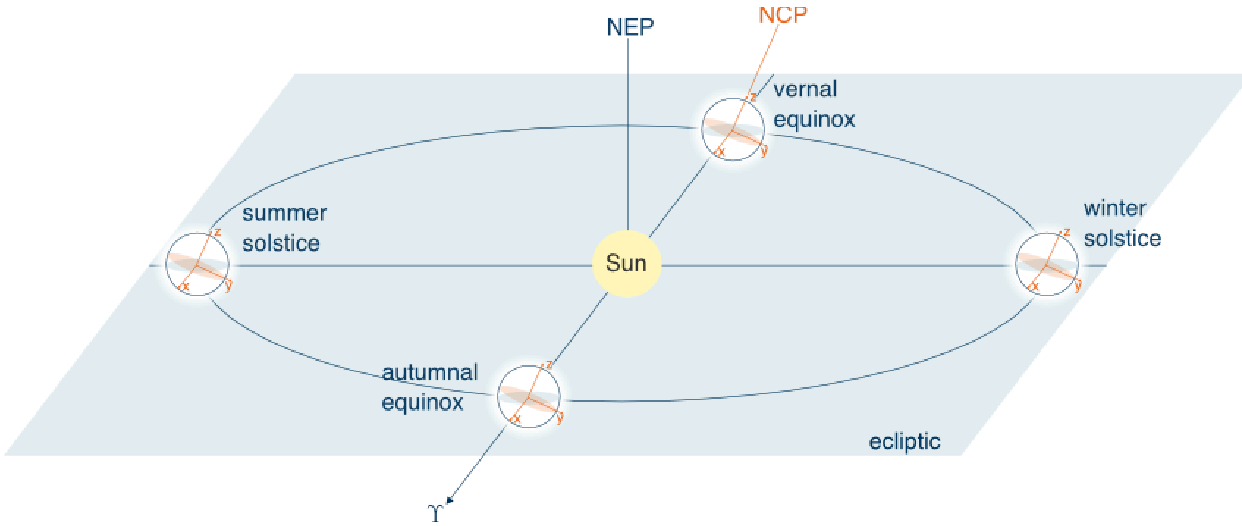

Figure 20 — Earth's rotation around the Sun

6.1. Space-fixed reference systems

A space-fixed system is the best approximation to an inertial system. Recall that any motion obeys Newton's laws.

$$m\ddot{\mathbf{r}} = \mathbf{F}(\mathbf{r}, \dot{\mathbf{r}}, t) \tag{1}$$

which states that the product of mass m and acceleration vector $\mathbf{a}$ is equal to the sum of all applied forces. In general, $\mathbf{F}$ depends on the location $\mathbf{r}$, the velocity $\dot{\mathbf{r}}$, and the time t. Practically, only approximations to an inertial or quasi-intertial system can be realized.

Triad vectors: e_{i0} with $i_0 = 1{,}2{,}3$
A space-fixed system is drawn up by a set of orthogonal, right-handed basis vectors. The zero in the index indicates a system defined at a fixed reference epoch. Here it will be generally referred to January 1st, 2000, 12:00 UT1 (J2000.0 for short). The *x*- or $e_{i0=1}$ -direction points to the mean vernal equinox at epoch J2000.0. The *z*- or $e_{i0=3}$ -axis defines the mean celestial pole, which currently points in the direction of Polaris ($\delta = 89.2°$). The *y*- or $e_{i0=2}$-axis completes the set of orthogonal basis vectors in a right-handed sense. The vectors $e_{i0=1}$ and $e_{i0=2}$ together span the plane of the celestial equator. A point, such as a star S, has the space-fixed spherical coordinates:

$$\mathbf{r}_S = \begin{pmatrix} \cos(\delta)\cos(\alpha) \\ \cos(\delta)\sin(\alpha) \\ \sin(\delta) \end{pmatrix} \tag{2}$$

The longitude coordinate on the celestial sphere is called right ascension α, counterclockwise from the vernal point and usually expressed in decimal hours (1h = 15°). The latitudinal coordinate is called declination δ and is expressed in degrees.

NOTEIf a space-fixed system is geocentric, it is often referred to as Earth-Centered Inertial (ECI).

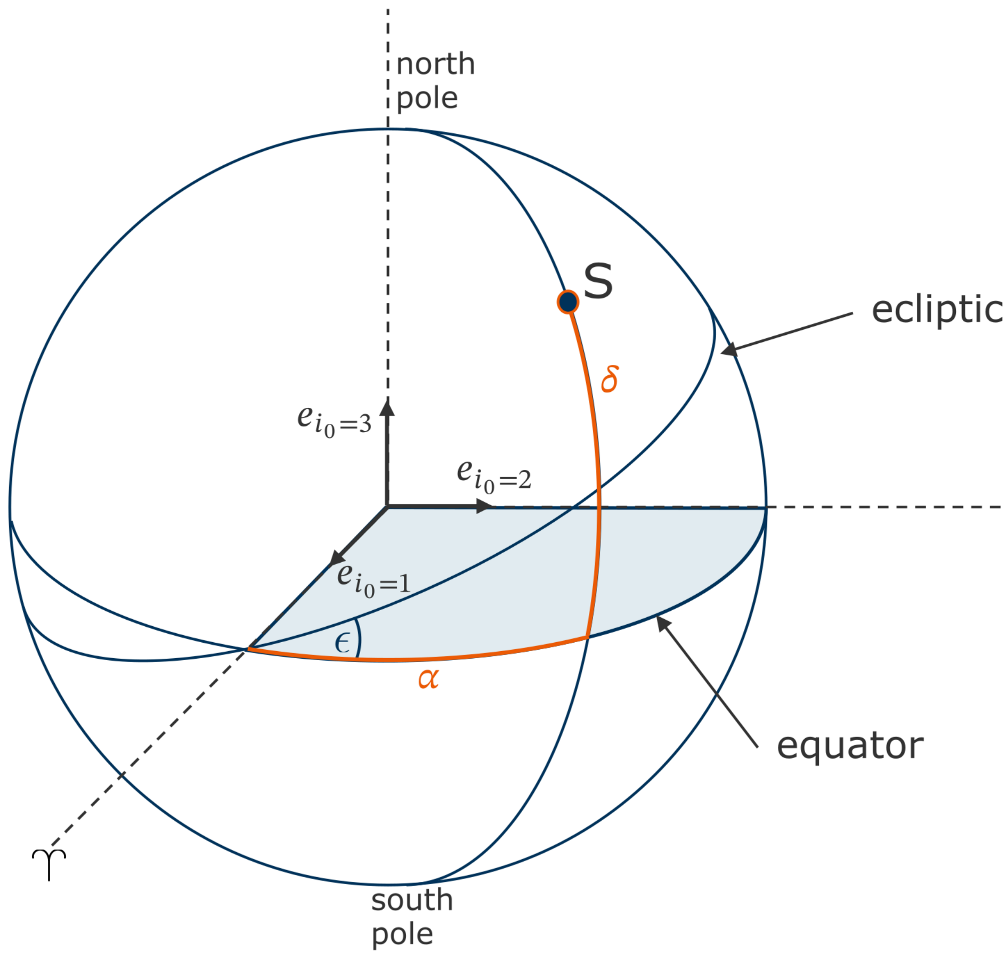

Figure 21 — Geocentric Space-fixed systems and space-fixed direction coordinates

6.1.1. Hierarchy within the space-fixed systems

Within the space-fixed reference system there are different approximations, in which the term “inertial” is defined more or less strictly.

6.1.1.1. Conventional inertial system

This system represents the best approximation to an inertial system. It is denoted by the index i_0 because it is defined at a fixed reference epoch T_0 and uses the mean orientation of the vernal equinox and the celestial pole at that time. The International Earth Rotation and Reference

Systems Service (IERS) takes a major role in defining reference systems in geodesy, referring to the conventional inertial system as the **International Celestial Reference System** (ICRS).
The triad is represented by e_{i_0} with $i_0 = 1,2,3$ as follows.

- $i_0 = 1$ $\overline{\Upsilon}(T_0)$ = mean vernal equinox at reference epoch T_0
- $i_0 = 2$ completes a right-handed system
- $i_0 = 3$ mean celestial pole at reference epoch T_0 (NCP_0)

6.1.1.2. Mean inertial system

This inertial system takes the precession motion into account and from the reference epoch T_0 to the epoch T. It is marked by the index $\bar{i}$. The bar above the *i* emphasizes the use of the mean orientation of the vernal equinox and the celestial pole to the epoch T.
The triad is represented by $e_{\bar{i}}$ with $\bar{i} = 1,2,3$ as follows.

- $\bar{i} = 1$ $\overline{\Upsilon}(T)$ = mean vernal equinox at reference epoch T
- $\bar{i} = 2$ completes a right-handed system
- $\bar{i} = 3$ mean celestial pole at epoch T (NCP_T)

6.1.1.3. True inertial system

In the true-of-date inertial system, in addition to the precession motion, the nutation motion is also considered. The true orientation of the vernal equinox and the celestial pole at epoch T are used.
The triad is represented by e_i with $i = 1,2,3$ as follows.

- $i = 1$ $\Upsilon(T)$ = true vernal point to the reference epoch T
- $i = 2$ completes a right-handed system
- $i = 3$ true celestial pole to the reference epoch T, which is called celestial ephemeris pole (CEP)

6.2. Earth-fixed reference systems

These systems are fixed to the Earth and of great importance for terrestrial applications. A complete definition usually also includes origin (such as center of mass), scale, and a reference ellipsoid. In the literature one often encounters the name Earth-Centered Earth-Fixed or ECEF-systems.

Triad vectors: e_e with $e = 1,2,3$

Similar to the space-fixed reference system, an Earth-fixed system is also drawn up by a set of orthogonal right-handed basis vectors. The *x*- or $e_{e=1}$-axis points to the meridian of Greenwich and lies within the equator plane. The *z*- or $e_{e=3}$-axis points to the north pole.

The *y*- or $e_{e=2}$-axis completes the right-handed system and defines with the vector $e_{e=1}$ the equator plane. The vectors $e_{e=1}$ and $e_{e=3}$ define the meridian plane through Greenwich.

Distinguishing between multiple ways to represent latitudes is important. The best known are geocentric ($\phi^{\text{geoc.}}$), geodetic ($\phi^{\text{geod.}}$), and astronomical (Φ), shown in Figure 23. The astronomical coordinates refer to the tangent to the true plumb line at the observation point. The perpendicular direction depends on the mass distribution of the Earth. In general, the plumb line is therefore irregularly curved and is neither perpendicular to the reference ellipsoid nor to the circle through the observation point.

Geodetic coordinates refer to the ellipsoidal normal, which is perpendicular to the surface of the reference ellipsoid. In doing so, the absolute position of the origin for different latitudes will vary.

In contrast to this, the geocentric coordinates all have the identical origin, which is the center of mass of the Earth.

The discrepancy between ellipsoid normal and plumb line is called deflection of the vertical (approx. 10^{-4} degree). The deflection of the vertical is divided into an east-west component η and a north-south ξ component. Geodetic measurements allow the determination of astronomical coordinates. When leveling a geodetic instrument, it is aligned to the local plumb line. When simultaneous geodetic coordinates are observed, the deflection of the vertical can be calculated via differencing.

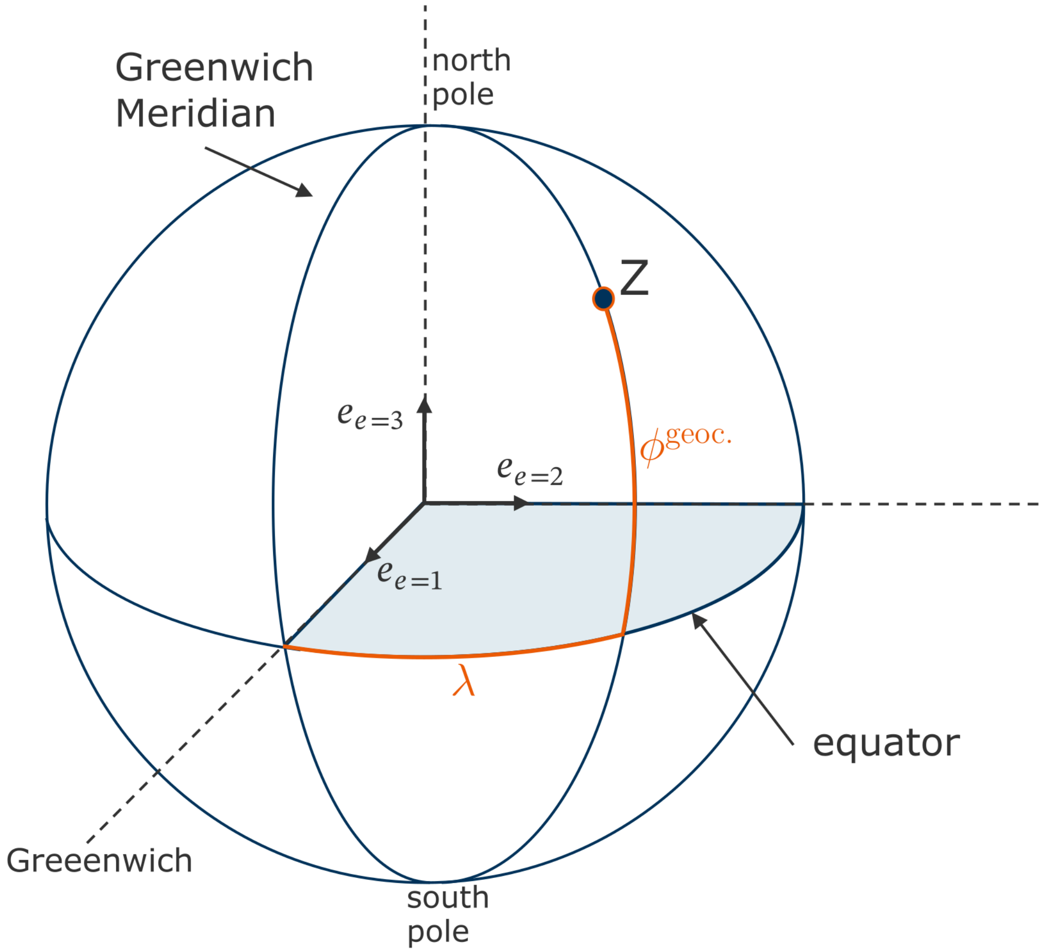

Figure 22 — Earth-fixed system with geocentric coordinates

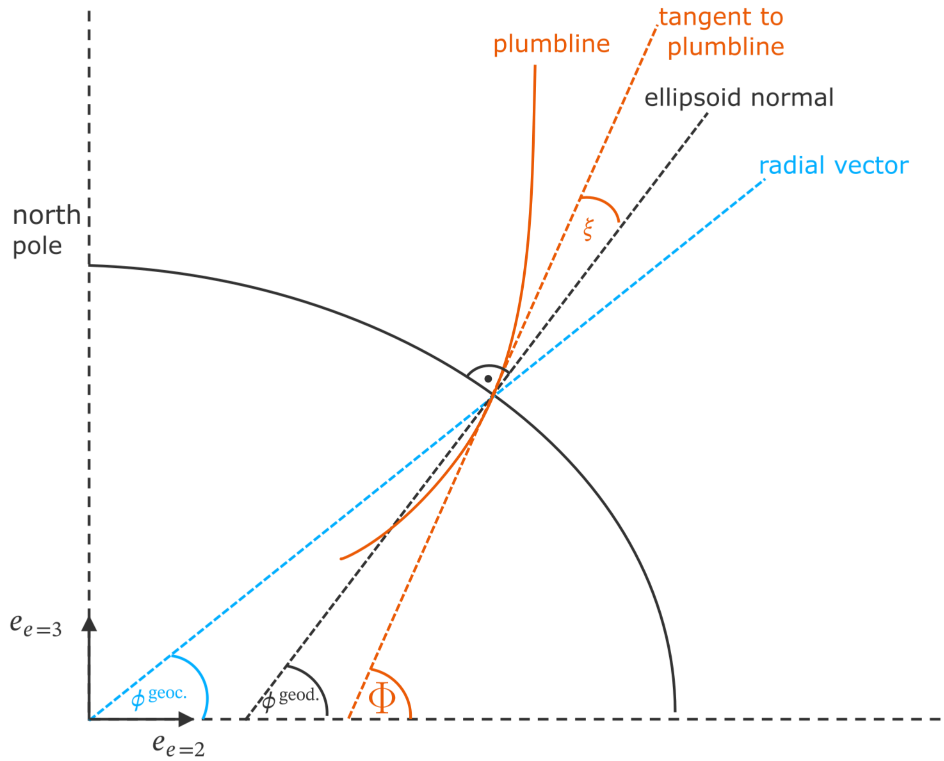

Figure 23 — Geocentric, geodetic and astronomical latitude

6.2.1. Hierarchy within the Earth-fixed system

Further, there are different gradations within the Earth-fixed reference systems. A true and a conventional terrestrial system is necessary to describe the position of the rotation axis, analogous to the inertial systems. These systems use cartesian coordinates, but in practice latitude, longitude, and height are preferred. Therefore, a third coordinate system is introduced with the use of a reference ellipsoid.

6.2.1.1. True terrestrial system

In the true terrestrial system, the position of the instantaneous rotation axis is described with respect to the conventional terrestrial system. This movement is also called polar motion. The true terrestrial system uses the true orientation of the celestial pole to the measuring epoch T. The triad is represented by e_e with $e = 1{,}2{,}3$ as follows.

- $e = 1$ conventional Greenwich meridian
- $e = 2$ completes a right-handed system
- $e = 3$ true celestial pole to the reference epoch T, which is called celestial ephemeris pole (CEP)

6.2.1.2. Conventional terrestrial system

This system forms the counterpart to the conventional inertial system. With the help of these two systems, the movement of the Earth axis can now be described exactly. The associated reference system by the IERS is called **International Terrestrial Reference System** (ITRS).
The triad is represented by e_{e_0} with $e_0 = 1,2,3$ as follows.

- $e_0 = 1$ conventional Greenwich meridian
- $e_0 = 2$ completes a right-handed system
- $e_0 = 3$ conventional north pole to reference epoch T_0

6.2.1.3. Reference ellipsoid / Geodetic terrestrial reference system

Geodetic coordinates are widely used in geodesy because of their ease of use. An ellipsoid is a better approximation to the Earth than a sphere. The origin of the geodetic terrestrial reference system is the mathematical center of the ellipsoid, which does not necessarily correspond to the center of mass of the Earth. The best known example for a geodetic terrestrial reference system is the **World Geodetic System 1984** (WGS84). In that case, the origin is equal to the center of mass of the Earth. When using the ISO 19111 Standard these systems are called geodetic.

6.3. Local systems

Local reference systems are mainly used to couple measurements of a geodetic (e.g., theodolite) or non-geodetic (e.g., telescope) instrument to Earth and space fixed reference systems. The horizontal and vertical axes of the instrument define the reference system. As a rule, they refer to the true plumb line (hence the index “g”), since the process of leveling aligns the instrument to the plumb line. It should be mentioned that there are also local reference systems which refer to the ellipsoid surface and the ellipsoid normal (often with index “γ”). In terms of ISO 19111 those systems are categorized in the *engineering* subtype.

Triad vectors: e_g with $g = 1,2,3$
A local system is defined up by a set of orthogonal, often left-handed basis vectors. The origin of the system is a point of interest and in geodesy often the location of the instrument. The x- or $e_{g=1}$ axis points north. The z- or $e_{g=3}$ axis points to the zenith. The second axis, y- or $e_{g=2}$ axis completes the left-handed systems and points to the west and together with the vector $e_{g=1}$ defines the horizontal plane. In a right-handed system the directions are similar. The first axis points to the north, the third to the zenith and the second axis completes a right-handed system and therefore points to the east. This system is often referred to as North-East-Up or NEU-system. In this ER only left-handed systems are discussed further. These systems can be converted into right-handed systems.

6.3.1. Hierarchy within the local systems

The local reference systems can also be subdivided. The differences arise from the choice of the north and zenith directions.

6.3.1.1. Local astronomical triad at Epoch T

This system can be derived from the true terrestrial system by rotation and translation with the astronomical coordinates (Φ, Λ, R). An example is where, in addition to the horizon, the north direction is determined from, e.g., star observations.
The triad is represented by e_g with $g = 1,2,3$ as follows.

- $g = 1$ true north-direction
- $g = 2$ completes a left or right-handed system
- $g = 3$ true zenith (tangent to true plumb line)

6.3.1.2. Local astronomical triad at Epoch T_0

The local astronomical system, unlike the previous system, uses the conventional north direction for the orientation of the $e_g = 1$ axis. It is therefore derived from the conventional terrestrial system by rotation and translation using the conventional astronomical coordinates (Φ_0, Λ_0, R_0). Measurements where the instrument is horizontal, but the north direction is determined from connecting to previous measurements, take place in this system.
The triad is represented by e_{g_0} with $g_0 = 1,2,3$ as follows.

- $g_0 = 1$ conventional north-direction
- $g_0 = 2$ completes a left or right-handed system
- $g_0 = 3$ true zenith (tangent to true plumb line)

6.3.1.3. Local geodetic system

This local geodetic system is a local system referring to the reference ellipsoid. Therefore, the normal and the north-direction of the ellipsoid are used.
The triad is represented by e_γ with $\gamma = 1,2,3$ as follows.

- $\gamma = 1$ conventional north-direction on ellipsoid
- $\gamma = 2$ completes a left or right-handed system

- $\gamma = 2$ local normal to ellipsoid

6.4. Reference frames

A reference system is the definition of a set of rules, not yet a collection of points and coordinates. The realization of these systems is called reference frame. The most important reference frames are described as follows.

6.4.1. International Celestial Reference Frame

This is the realization of the ICRS by the IERS. The International Celestial Reference Frame (ICRF) is defined by the coordinates of over 600 extraterrestrial points that have been observed by Very Long Baseline Interferometry (VLBI) in J2000. The position of the quasars, which are extragalactic radio sources, is determined by their right ascension α and declination δ. Thus, the ICRF is a realization in the radio frequency band.
Classically, star coordinates have been measured in the optical waveband. This has resulted in a series of fundamental catalogues, such as FK5. Due to atmospheric refraction, these coordinates cannot compete with VLBI-derived coordinates. However, in the early 1990s, the astrometry satellite HIPPARCOS collected the coordinates of over 100 000 stars with a precision better than 1 milliarcsecond. The HIPPARCOS catalogue constitutes the primary realization of an inertial frame at optical wavelengths. Multiple versions of the ICRF exist, with the latest version being ICRF 3 from the year 2020.

6.4.2. International Terrestrial Reference Frame

In addition to the conventional inertial reference system (ICRS), the IERS also defines a conventional terrestrial reference system, the International Terrestrial Reference System (ITRS). The realization of this is the International Terrestrial Reference Frame (ITRF). The ITRF is updated in irregular intervals, with ITRF2020 being the most current version.
The transformation between different frames of the ITRF is stated as a 7-parameter transformation. Of these, three are parameters for translation, three for rotation, and one is a scale factor. Since the rotation angles between different frames are very small, one approximates $\sin(x) = x$.

$$\mathbf{X}_2 = \mathbf{X}_1 + \mathbf{T} + DX_1 + \mathbf{R}X_1$$
$$\mathbf{T} = \begin{pmatrix} T1 \\ T2 \\ T3 \end{pmatrix} \quad \text{and} \quad \mathbf{R} = \begin{pmatrix} 0 & -R3 & R2 \\ R3 & 0 & -R1 \\ -R2 & R1 & 0 \end{pmatrix} \tag{3}$$

The latest frames additionally contain ranges of change, i.e., coordinate velocities, resulting in a total of 14 parameters. All parameters, therefore, are a function of time.

$$\dot{\mathbf{X}}_2 = \dot{\mathbf{X}}_1 + \dot{\mathbf{T}} + \dot{D}\mathbf{X}_1 + D\dot{\mathbf{X}}_1 + \dot{\mathbf{R}}\mathbf{X}_1 + \mathbf{R}\dot{\mathbf{X}}_1 \tag{4}$$

6.4.3. World Geodetic System 1984

The World Geodetic System (WGS84) is perhaps the best known reference frame, especially since it is used in Global Navigation Satellite Systems such as GPS. WGS84 itself defines a reference system, with multiple realizations, done with Doppler, SLR, VLBI, and GNSS. The transformation between ITRF and WGS84 is performed in a similar fashion to the transformation between two ITRF versions. The parameters can be found in literature (Handbook of GNSS) or online.

7

TRANSFORMATIONS BETWEEN INERTIAL AND TERRESTRIAL REFERENCE SYSTEMS

7 TRANSFORMATIONS BETWEEN INERTIAL AND TERRESTRIAL REFERENCE SYSTEMS

Given that the hierarchy of coordinate systems has been discussed, a description of the transformations and conversions between the systems follows.

7.1. Conventional inertial ←→ true inertial

First, the two transformations due to precession and nutation are described. An older convention requires that two separate transformations be performed in succession. According to a new convention, however, these can be performed in a single transformation because in most cases both phenomena should be considered.

7.1.1. Precession

This transformation from the conventional inertial reference system at epoch T_0 to the mean inertial one at epoch T describes the phenomenon of precession. Simon Newcomb (1835–1909), formulated the transition of the coordinates r as follows.

$$\mathbf{r}_{\bar{i}} = \mathbf{P}\mathbf{r}_{i_0} = \mathbf{R}_3(-z)\mathbf{R}_2(\theta)\mathbf{R}_3(-\zeta_0)\mathbf{r}_{i_0} \quad (5)$$

First, a rotation around the north celestial pole at epoch T_0 (NCP_0) shifts the mean equinox at epoch T_0 over the mean equator at T_0. This is $R_3(-\zeta_0)$. Next, the NCP_0 is shifted along the cone towards the mean pole at epoch T (NCP). This is a rotation $R_2(\theta)$, which also brings the mean equator at epoch T_0 is brought to the mean equator at epoch T. Finally, a last rotation around the new pole, $R_3(-z)$ brings the mean equinox at epoch T ($\bar{\Upsilon}_T$) back to the ecliptic.
The required precession angles are given with a precision of 1” using the following.

$$\zeta_0 = 2.650545'' + 2{,}306.083227''T + 0.2988499''T^2 + 0.01801828''T^3 - 0.5971'' \cdot 10^{-6}T^4 - 3.173'' \cdot 10^{-7}T^5$$
$$z = -2.6505453'' + 2{,}306.0771813''T + 1.0927348\beta''T^2 + 0.018268373''T^3 - 28.596'' \cdot 10^{-6}T^4 - 2.904'' \cdot 10^{-7}T^5 \quad (6)$$
$$\theta = 2{,}004.191903''T - 0.4294934''T^2 - 0.041822''T^3 - 7.089'' \cdot 10^{-6}T^4 - 1.274'' \cdot 10^{-7}T^5$$

[**SOURCE:** Handbook of GNSS]

The time T is counted in Julian centuries (of 36,525 days) since J2000.0, i.e., January 1, 2000, 12^h UT1. The time in Julian centuries is calculated from calendar date and universal time (UT1) by first converting to the so-called *Julian day* number (JD), which is a continuous count of the number of days. A detailed discussion of time systems will follows later in this ER, for more information on UT1 see Clause 8.

7.1.2. Nutation

The following transformation describes the transition from the mean inertial reference system (MOD) to the true inertial (TOD) one. This transformation deals with the phenomenon of nutation.
The mathematical expression for this is:

$$\mathbf{r}_i = \mathbf{N}\mathbf{r}_{\bar{i}} = \mathbf{R}_1(-\varepsilon - \Delta\varepsilon)\mathbf{R}_3(-\Delta\psi)\mathbf{R}_1(\varepsilon)\mathbf{r}_{\bar{i}} \quad (7)$$

First, the mean equator at epoch T is rotated into the ecliptic around $\bar{\Upsilon}_T$. This rotation, $R_1(\varepsilon)$, brings the mean north pole towards the NEP. Next, a rotation $R_3(-\Delta\psi)$ brings the mean equinox over the ecliptic towards the true inertial epoch. Finally, the rotation $R_1(-\varepsilon - \Delta\varepsilon)$ transforms back to an equatorial system.
The nutation angles are known as nutation in obliquity ($\Delta\varepsilon$) and nutation in (ecliptical) longitude ($\Delta\psi$). Together with the obliquity ε itself, which is minimally time-dependent.

$$\varepsilon = 84{,}381.448'' - 46.8150''T \quad (8)$$

The obliquity ε is given in arcseconds. Converted into degrees it equals $\varepsilon \approx 23.4^\circ$. It also changes by some 47” per Julian century. The nutation angles are not exact. They are often realized using a Fourier series:

$$\begin{aligned}\Delta\psi &= \sum_{i=1}^{n}\left(a_i \sin A_i + a_i' \cos A_i\right)\\ \Delta\varepsilon &= \sum_{i=1}^{n}\left(b_i \cos A_i + b_i' \sin A_i\right)\end{aligned} \quad (9)$$

using $A_i = n_{l,i} l + n_{l',i} l' + n_{F,i} F + n_{D,i} D + n_{\Omega,i}\,\Omega$

which represents a linear combination of fundamental arguments of solar and lunar orbits.

l Mean anomaly of the Moon

l' Mean anomaly of the Sun

F Mean longitude of the Moon minus the mean longitude of the Moon’s ascending node

D Mean elongation of the Moon from the Sun

Ω Mean longitude of the ascending node of the Moon

The coefficients can be taken from the Observatoire de Paris or the IERS. Only regarding the two main frequencies, while still calculating the angles with precision of 1”, the formula can be simplified to:

$$\begin{aligned}\Delta\varepsilon &= 0.0026^\circ\cos(f_1) + 0.0002^\circ\cos(f_2)\\ \Delta\psi &= -0.0048^\circ\sin(f_1) - 0.0004^\circ\sin(f_2)\end{aligned} \quad (10)$$

with $f_1 = 125.0^\circ - 0.05295^\circ d$ and $f_2 = 200.9^\circ + 1.97129^\circ d$. The coefficients to the variable d are frequencies in units of degree/day so f_1 corresponds to a period of 18.6 years and f_2 to a semiannual period.

7.1.3. New convention

The first realization of the transformation regarding a conventional inertial and a true inertial system would be to simply concatenate the two transformations of precession and nutation:

$$\mathbf{r}_i = \mathbf{NP}\mathbf{r}_{i_0} \tag{11}$$

Another approach is a convention defined by the IERS in 2010. In this approach both the precession and nutation are taken into account, reducing the calculation to three instead of six rotations:

$$\mathbf{r}_i = \mathbf{R}_3(-s)\mathbf{R}_3(-E)\mathbf{R}_2(d)\mathbf{R}_3(E)\mathbf{r}_{i_0} = \mathbf{Q}^\top \mathbf{r}_{i_0} \tag{12}$$

For this transformation the location of the true or instantaneous pole is described via the co-declination, d, and the right ascension, E, with respect to the reference pole. The following is a conversion into cartesian coordinates:

$$\begin{pmatrix} X \\ Y \\ Z \end{pmatrix} = \begin{pmatrix} \sin(d)\cos(E) \\ \sin(d)\sin(E) \\ \cos(d) \end{pmatrix} \tag{13}$$

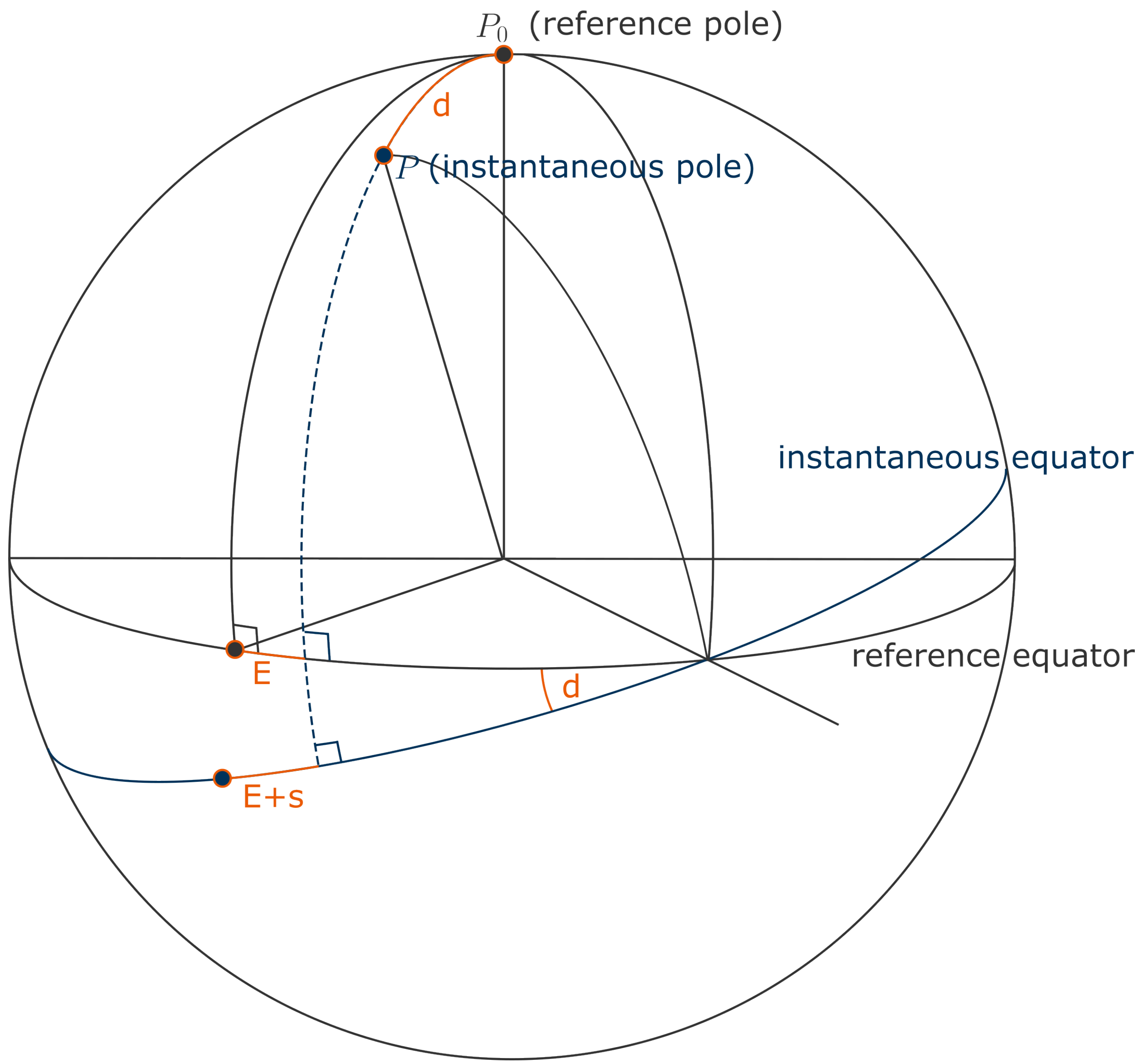

Figure 24 — Coordinates of the instantaneous pole in the celestial reference system

The matrix Q which represents the transformation can be rewritten to:

$$\mathbf{Q} = \begin{pmatrix} 1 - aX^2 & -aXY & X \\ -aXY & 1 - aY^2 & Y \\ -X & -Y & 1 - a(X^2 + Y^2) \end{pmatrix} \mathbf{R}_3(s) \quad (14)$$

with $a = \frac{1}{1 + \cos(d)}$, which is, with an accuracy of 1 µas (micro-arcsecond), equal to $a = \frac{1}{2} + \frac{1}{8}(X^2 + Y^2)$.

The coordinates X, Y and the last rotation s are calculated with the following formula:

$$s = -\tfrac{1}{2}XY + 94 + 3{,}808.65T - 122.68T^2 - 72{,}574.11T^3 + \sum_k C_k \sin\alpha_k + \sum_k D_k \sin\beta_k + \sum_k E_k T\cos\gamma_k + \sum_k F_k T^2 \sin\theta_k \, (\mu as)$$

$$X = -0.016617'' + 2{,}004.191898''T - 0.4297829''T^2 - 0.19861834''T^3 - 0.000007578''T^4 - 0.0000059285''T^5 + \sum_{i=1}^{n}\left(e_i \sin A_i + e_i' \cos A_i\right) \quad (15)$$

$$Y = -0.006951'' - 0.025896''T - 22.4072747''T^2 + 0.00190059''T^3 + 0.001112526''T^4 - 0.0000001358''T^5 + \sum_{i=1}^{n}\left(f_i \sin A_i + f_i' \cos A_i\right)$$

[**SOURCE:** Handbook of GNSS]

7.2. True inertial ←→ true terrestrial

For the transformation from the true inertial reference system to the true terrestrial system the transformation only brings the true equinox to the Greenwich meridian. The angle between the x-axes of both systems is the Greenwich Actual Sidereal Time **GAST**. A more precise definition of this angle is given in Clause 8.
Thus, the following rotation is required for the transformation:

$$\mathbf{r}_e = \mathbf{R}_3(\mathrm{GAST})\,\mathbf{r}_i \quad (16)$$

The angle GAST is calculated from the Greenwich Mean Sidereal Time (GMST) by applying a correction for the nutation:

$$\mathrm{GMST} = \mathrm{UT1} + \frac{24{,}110.54841 + 8{,}640{,}184.812866T + 0.093104T^2 - 6.210^{-6}T^3}{3{,}600} + 24n$$
$$GAST = GMST + \frac{\Delta\psi\cos(\varepsilon + \Delta\varepsilon)}{15} \quad (17)$$

Universal time UT1 is in decimal hours and n is an arbitrary integer that makes 0 ≤ GMST < 24. The nutation correction from GMST to GAST is the so called equation of equinoxes (Eq.E.).

7.3. True terrestrial ←→ conventional terrestrial

The following transformation describes the transition from the true terrestrial system to the conventional terrestrial one, which is the correction for **polar motion**.
To correct for polar motion the Conventional International Origin (CIO) is defined as the mean pole of the years 1900-1905 measured by the International Latitude Service. A translation on the surface by $x_\mathrm{p}, y_\mathrm{p}$ leads to the instantaneous pole defined by the z_e-axis. The axis through the CIO is the z_{e_0}-axis of the conventional terrestrial system e_0.

The transformation from instantaneous (true) terrestrial to the conventional terrestrial system reads:

$$\mathbf{r}_{e_0} = \mathbf{R}_2(-x_p)\mathbf{R}_1(-y_p)\mathbf{r}_e \tag{18}$$

x_p and y_p are derived from observations of the International Earth Rotation and Reference Systems Service (IERS).
Written as differential rotations, this transformation can be expressed by:

$$\mathbf{r}_{e_0} = \begin{pmatrix} 1 & 0 & x_p \\ 0 & 1 & -y_p \\ -x_p & y_p & 1 \end{pmatrix} \mathbf{r}_e \tag{19}$$

7.4. Geodetic terrestrial → local geodetic

The transformation from the geodetic terrestrial reference system to the local geodetic system is often used to transform into the coordinate system of the object / area of interest. The transformation states:

$$\mathbf{r}_\gamma = \mathbf{P}_1\mathbf{R}_2(\theta)\mathbf{R}_3(\lambda)(\mathbf{r}_\varepsilon - \mathbf{r}_{\gamma,0})$$
$$\text{using } \theta = 90° - \phi \text{ and } \mathbf{P}_1 = \begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{20}$$

The vector $r_{\gamma,0}$ describes the origin of the local system in e-coordinates, and thus the translation. It is important to know whether geodetic or geocentric latitudes are used (see Figure 23). The rotation sequence remains the same, as shown in Figure 25.

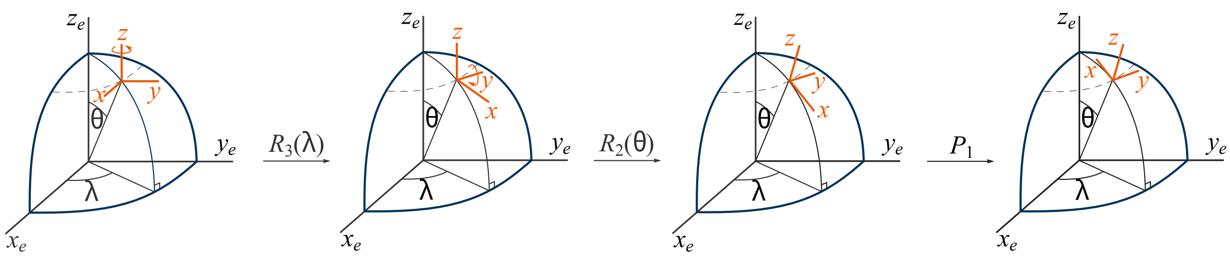

Figure 25 — Rotations from geodetic terrestrial to local geodetic reference systems

8

TIME SYSTEMS

8 TIME SYSTEMS

Time is the fourth coordinate in four-dimensional space-time. As with the coordinate systems discussed in the previous chapters, it is necessary to discuss concepts like origin, scale, time-evolution, and the distinction between a system and its realization (a frame). Moreover, the three space coordinates are strongly interwoven with time. For example, the unit of length itself, the meter, is defined in terms of the amount of time it takes a light wave to travel through vacuum. Also the transformation between inertial and terrestrial coordinate systems requires the angle GAST, the Greenwich Apparent Sidereal Time. The Earth rotation is $360^\circ/\mathrm{day}$, which results in a linear velocity of about $450\,\mathrm{m}/\mathrm{s}$ at the equator.

8.1. Preliminary considerations

The word *time* can be understood in three senses. First, it means *epoch*, which is an instant, or a point in time. One can speak of the epoch of a GPS measurement. Second, time can be understood as an *interval*, which is the difference between two epochs. The third sense is time *scales*, which is the division of an interval into time units. The following are four classes of time systems and the transformations between them.

1. **sidereal time** which refers to the stars
2. **solar or universal time** which refers to the Sun
3. **atomic time** which refers to atomic phenomena
4. **theoretical time** which refers to a theoretical model

The first two categories are natural times which describe the rotation state of the Earth in space. The third time system describes a physical phenomenon, namely oscillations of atoms or molecules. The last time system describes time as a theoretical state which can vary in dependence of velocity and gravitational potential.

Each category is described in this chapter followed by a short discussion on *calendar* dates and *Julian Day numbers*. An overview of time systems is given in Figure 26.

There are several criteria for time systems, which may partly be contradictory.

1. The time scale should be stable. In other words, a second now should last exactly as long as it lasted yesterday. There should preferably be no drift or periodic effects in the time definition. Because the Earth's rotation is gradually slowing down, this criterion is hard to meet in the long term.
2. The time scale should be accessible. It should not be necessary, for instance, to perform complicated astronomic observations to get a time reading.

3. The time scale should still be available or accessible over the long term. The astronomical observation of the Babylonians can still be used. On the other hand, it is doubtful whether modern highly precise atomic clocks are of any use to future civilizations.

4. For many purposes, the time system should be physically meaningful. For those purposes natural time is the preferable time system.

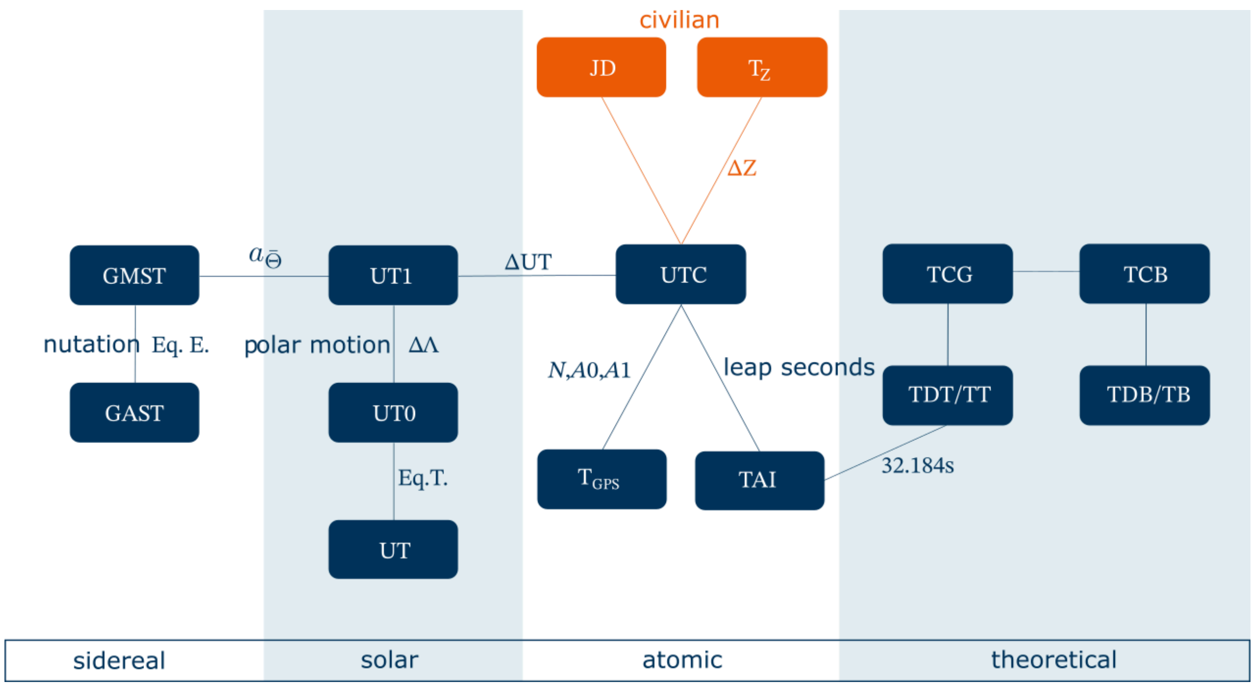

Figure 26 — Overview of time systems

8.2. Sidereal time

Sidereal time is the hour angle in the equator plane between a given meridian and the vernal equinox ♈#. This angle is conventionally expressed in units of hours, of which there are 24 in a full circle: $1^{\mathrm{h}} = 15°$. According to this definition, sidereal time describes the orientation of the Earth in inertial space. One sidereal day is the interval between two consecutive transits of the equinox (or of any star) through the meridian. This corresponds to a full revolution of the Earth around its axis.

If the true equinox Υ_T is taken as a reference, then the term *Apparent Sidereal Time* (AST) is used. Using the mean equinox $\bar{\Upsilon}_T$ results in *Mean Sidereal Time* (MST). If a particular meridian is the observer's local meridian, this is termed *Local Sidereal Time*. On the other hand, the angle between Greenwich and the equinox is the *Greenwich Sidereal time*. All 4 potential combinations are summarized in Table 1:

Table 1 — Sidereal time

	LOCAL MERIDIAN	GREENWICH
true equinox	LAST	GAST
mean equinox	LMST	GMST

8.2.1. LAST

A description of sidereal time begins with the fundamental astronomical triangle on the celestial sphere, cf. Figure 27. This figure shows LAST as an angle between local meridian (the x-axis of the hour angle system) and true equinox (the x-axis of the instantaneous inertial system). Moreover, by including the hour circle of a given star,this figure relates LAST to the hour angle h and the right ascension α:

$$\mathrm{LAST} = \alpha + h \tag{21}$$

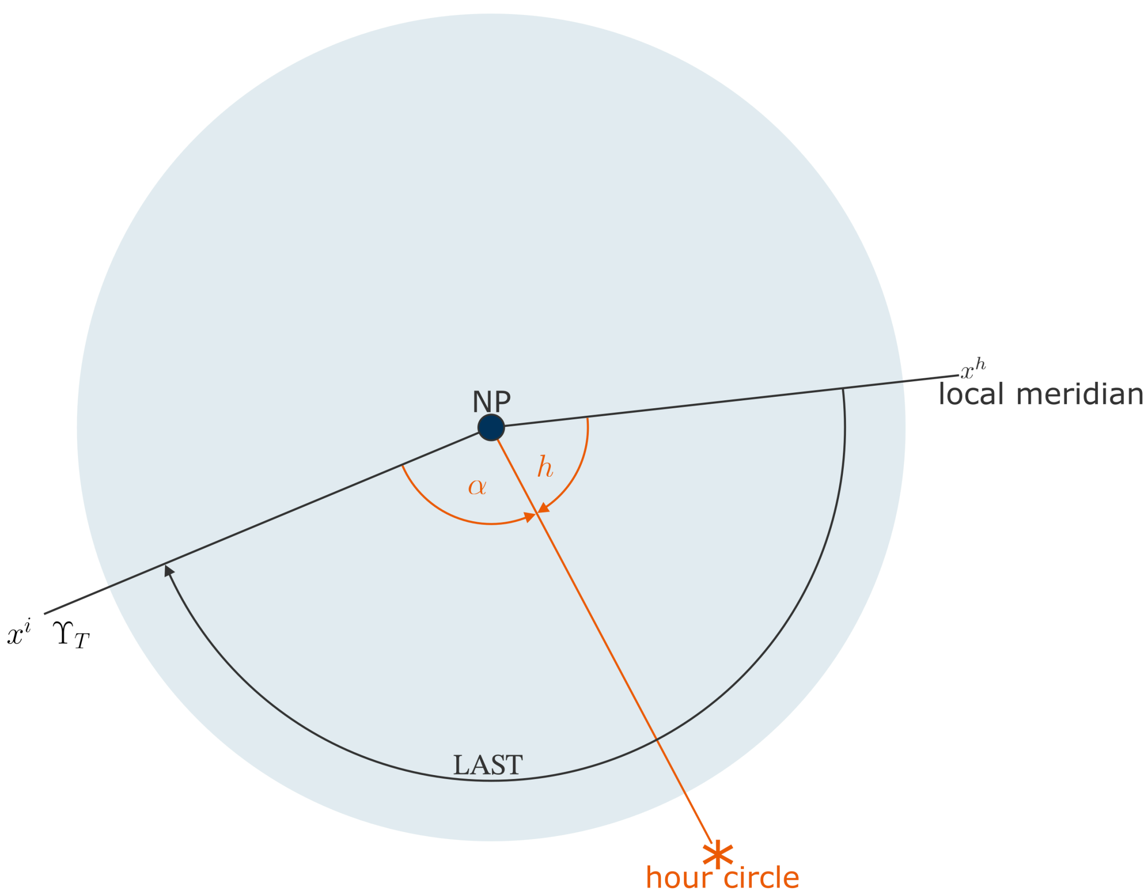

Figure 27 — Projection of the fundamental astronomical triangle on the equator plane

8.2.2. GAST

In Figure 28, the Greenwich meridian is taken instead of the local meridian. The difference between them is the astronomical longitude Λ. This results in the simple but fundamental relation:

$$\text{GAST} = \text{LAST} - \Lambda = \alpha + h - \Lambda \quad (22)$$

which says that time and longitude are intimately connected. It is possible to obtain astronomical longitude from the combined measurement of time (through GAST) and observation to a given star (through h), using the star's right ascension α from a catalogue. This observation is even simpler for stars passing the local meridian, in which case $h = 0$ applies. Such transits are called *upper culmination* if the star passes between zenith and north pole. A transit below the pole star is known as *lower culmination*.

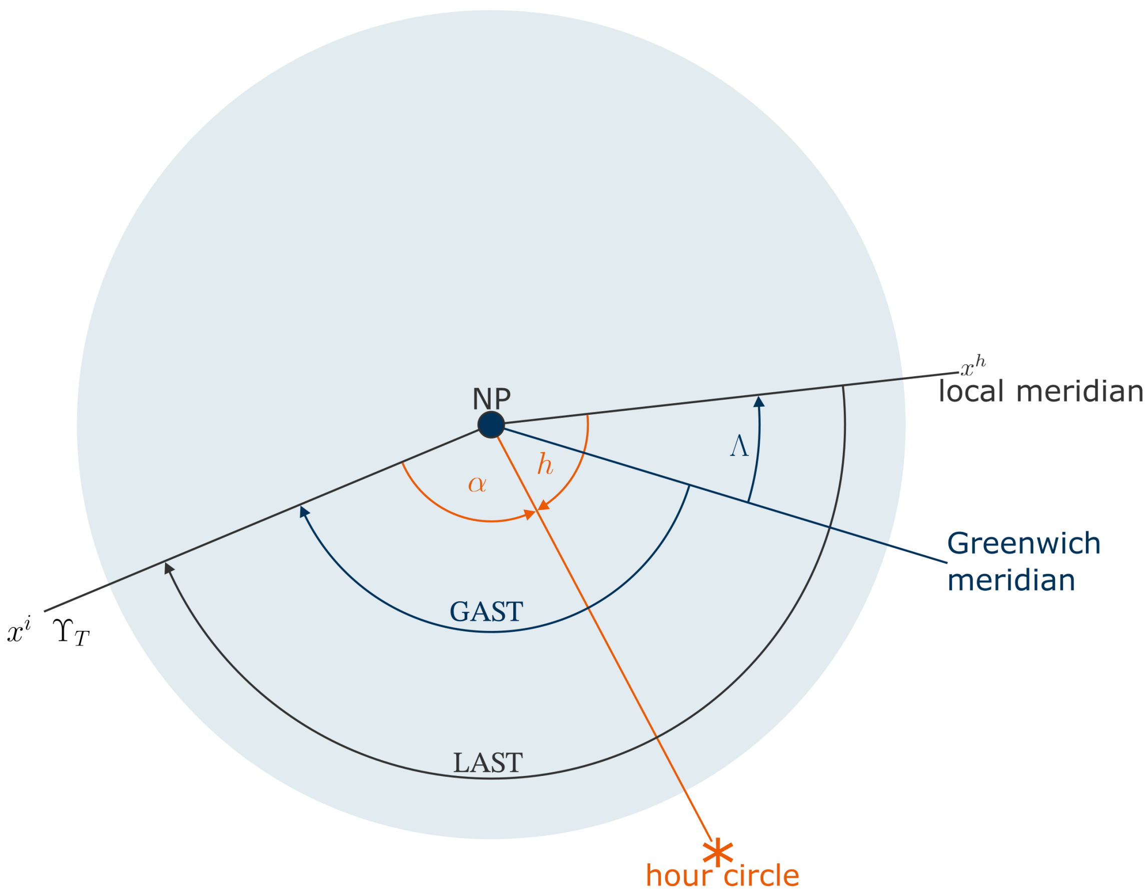

Figure 28 — Comparison of LAST and GAST

8.2.3. Comparison of LMST and GMST

One of the criteria for a useful time system is the stability of the time scale. This stability is not given in case of actual sidereal time. Because of nutation, the equinox will move back and forth over the ecliptic by the angle $\Delta\psi$, the nutation in longitude. The angles LAST and GAST are therefore relative to a time dependent reference point.

The circles drawn so far represent the equator. To correct for nutation, $\Delta\psi$ is projected on the equator. With the obliquity ϵ between ecliptic and equator, this projection becomes $\Delta\psi \cos\epsilon$. This is called the *Equation of the Equinox* or Eq.E:

$$\mathrm{Eq.E.} = \Delta\,\psi \cos(\,\varepsilon\,) = \mathrm{LAST} - \mathrm{LMST} = \mathrm{GAST} - \mathrm{GMST} \tag{23}$$

The variability in the Equation of the Equinox is only of the order of magnitude of roughly $1\,\mathrm{s}$. However, for a stable definition of a sidereal time system this is relevant.

One sidereal day is given as the time interval between two successive crossings of the mean equinox $\bar{\Upsilon}_T$ through the local meridian. Such a crossing, during which $\mathrm{LMST} = 0$ is the sidereal noon. One might subdivide the sidereal day in 24 sidereal hours of 60 sidereal minutes of 60 sidereal seconds. It is important to emphasize the word *sidereal* here, since these time units are slightly different from the solar equivalents discussed in the next section ny a factor F.

All sidereal time angles are presented in Figure 29, from which the following formulas are derived:

$$\begin{aligned} \mathrm{LAST} - \mathrm{GAST} = \mathrm{LMST} - \mathrm{GMST} &= \Lambda \\ \mathrm{LAST} - \mathrm{LMST} = \mathrm{GAST} - \mathrm{GMST} &= \mathrm{Eq.E} \\ \mathrm{LAST} &= \alpha + h \end{aligned} \tag{24}$$

NOTENote that the mean sidereal time refers to the mean equinox. However, this mean equinox at Epoch T is affected by precession. The mean equinox slides nearly uniformly over the ecliptic. For this reason, the stability of the mean sidereal times as defined here is not endangered. It does mean, though, that a mean sidereal day is $0.0084\,\mathrm{s}$ shorter than a full revolution of the Earth in a conventional inertial system.

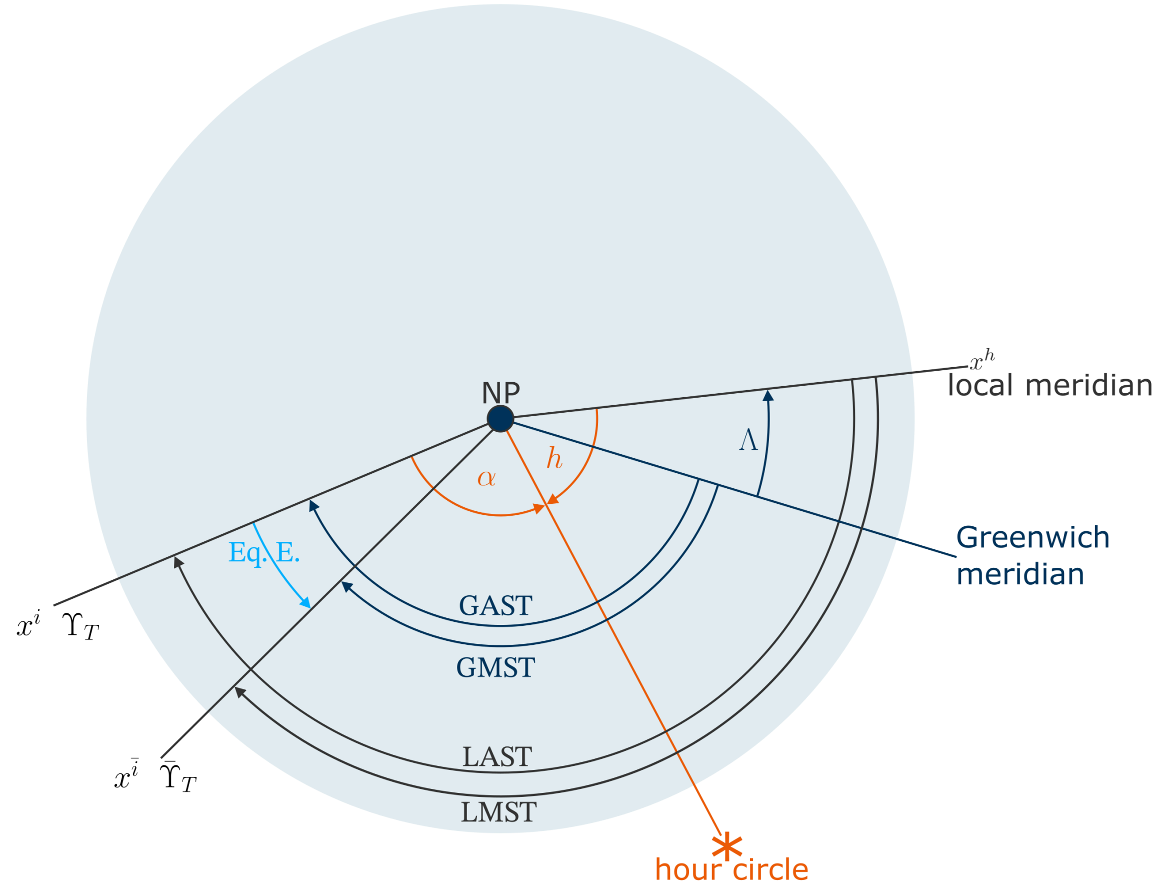

Figure 29 — The four basic types of sidereal time.

8.3. Solar or Universal time

A civilian time system must be based on the motion of the Sun, which therefore will be taken as a reference, instead of the equinox. Intuitively one associates noon (12^h) to the transition of the Sun through the local meridian. This intuitive solar time definition will, however, require some refinement.

One solar day is the time span between two successive meridian transits of the Sun. This interval is divided in 24 (solar) hours of 60 minutes of 60 seconds. Opposed to a sidereal day, however, one solar day does *not* correspond to a full revolution of the Earth around its axis. Since the Earth moves around the sun once a year, one solar day is consequently slightly more than a full revolution. Figure 30 again shows the difference figuratively. A year consists of 365.242 solar days. After one solar day, the Earth has traveled on average $\frac{360^\circ}{365.242\,\mathrm{d}}$ of its orbit around the Sun. This corresponds to $3^{\mathrm{m}}56^{\mathrm{s}}.33$ per day:

$$1 \text{ mean solar day} = 1 \text{ mean sidereal day} + 3^{\mathrm{m}}56^{\mathrm{s}}.33$$

Consequently, a solar second is longer than a sidereal second. The scale factor between these time scales can be determined by considering that the number of sidereal days within a year is exactly one more than the number of solar days. Therefore:

$$\text{scale factor:} \quad F = \frac{1 \text{ solar day}}{1 \text{ sidereal day}} = \frac{366.242}{365.242} = 1.002\,737\,91 \tag{25}$$

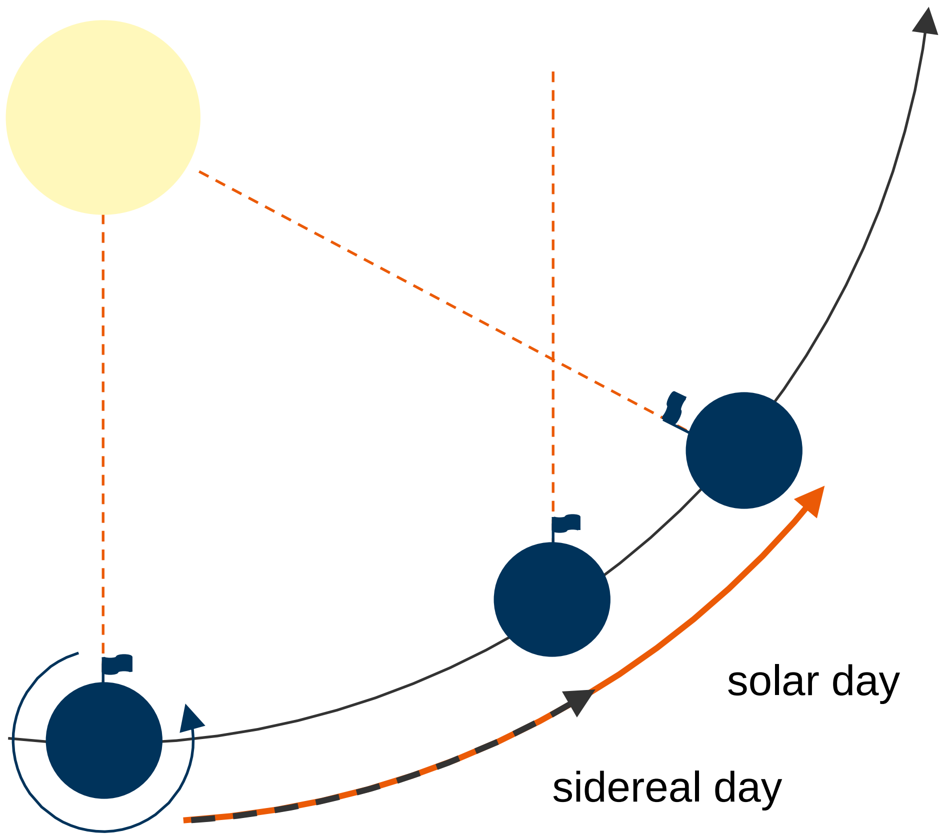

Figure 30 — Difference of sidereal and solar day relative to the sun.

8.3.1. LT, UT

Figure 31 graphically explains the basic definition of (LT) where LT is the hour angle of the Sun ⊙, i.e., the angle between local and solar meridian. However, since noon is associated with a zero-hour angle, 12^{h} are added:

$$\mathrm{LT} = h_{\odot} \pm 12^{\mathrm{h}} \tag{26}$$

Similarly, by taking the Greenwich meridian, results are in Greenwich solar time. This is known as true *universal time* (UT). It is called *true* because it refers to the real Sun:

$$\mathrm{UT} = h_{\odot}^{\mathrm{Gr.}} \pm 12^{\mathrm{h}} \tag{27}$$

Local and Greenwich solar time are simply related by adding or subtracting the longitude:

$$\mathrm{LT} = \mathrm{UT} + \Lambda \tag{28}$$

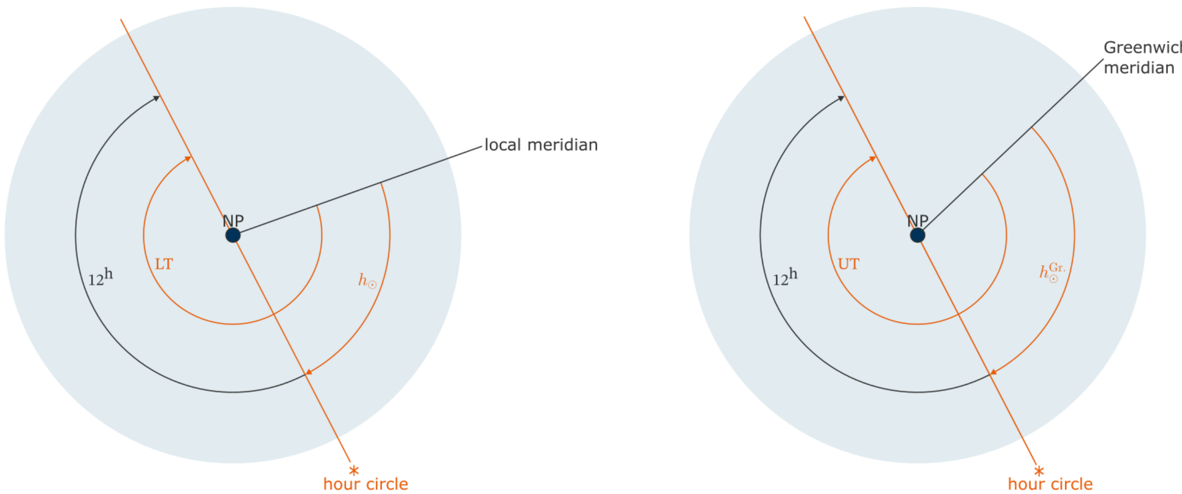

Figure 31 — The basic local (left) and Greenwich (right) solar time

8.3.2. Refinement 1: UT0

Similar to the true equinox, which was not suitable to define a stable sidereal time system, the motion of the true Sun is not homogeneous enough to define a stable solar time system. The reason for the non-uniform motion is twofold:

1. Ellipticity of the Earth's orbit around the Sun (according to Kepler's area law, the Earth moves faster near the perihelion (the point closest to the Sun)); and

2. Obliquity between equator and ecliptic plane.

Therefore, a fictitious mean Sun ($\bar{\odot}$) is introduced that moves along the equator with uniform speed. The difference between the true and fictitious Sun, projected onto the equator, is called *Equation of Time*:

$$\text{Eq.T.} = \alpha_{\bar{\odot}} - \alpha_{\odot} = h_{\odot} - h_{\bar{\odot}} \quad (29)$$

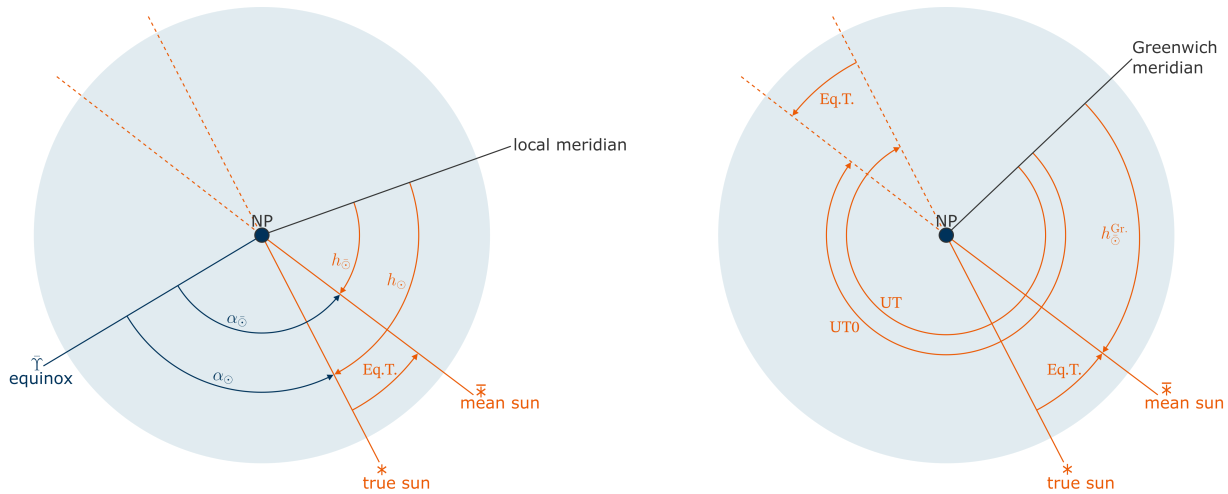

Figure 32 — Equation of Time (left) and definition of UT0 (right)

The Equation of Time can reach values of up to 15 minutes.

NOTEThe equation of time is often visualized by the so-called *analemma*. This is an 8-shaped curve, which is attained by a parametric plot of $x = \mathrm{Eq.T.}$ versus the solar declination $y = \delta_\odot$. The solar declination varies between the tropics ($\pm\epsilon$) through the year.

Employing the mean Sun, the first refinement to UT consists of correcting for the inhomogeneous solar motion. The result is called UT0:

$$\mathrm{UT0} = \mathrm{UT} - \mathrm{Eq.T.} = h_{\bar\odot}^{\mathrm{Gr.}} \pm 12^{\mathrm{h}} \quad (30)$$

8.3.3. Refinement 2: UT1

Due to polar motion, the instantaneous longitude of any meridian and therefore for this variability, the conventional terrestrial pole, is referred to the corresponding solar time for the conventional Greenwich meridian, which is called *Greenwich mean time* or UT1:

$$\begin{aligned} \mathrm{UT1} &= \mathrm{UT0} + \Delta\ \Lambda_{\mathrm{p}} \\ &= \mathrm{UT0} - \left(x_{\mathrm{p}} \sin\Lambda + y_{\mathrm{p}} \cos\Lambda\right) \tan\Phi \\ &= h_{\bar\odot}^{\mathrm{Gr.}} \pm 12^{\mathrm{h}} \end{aligned}$$

The result is a relatively stable time system for civilian time-keeping purposes, based on a physical and observable phenomenon: the orientation of the Earth with respect to the mean Sun.

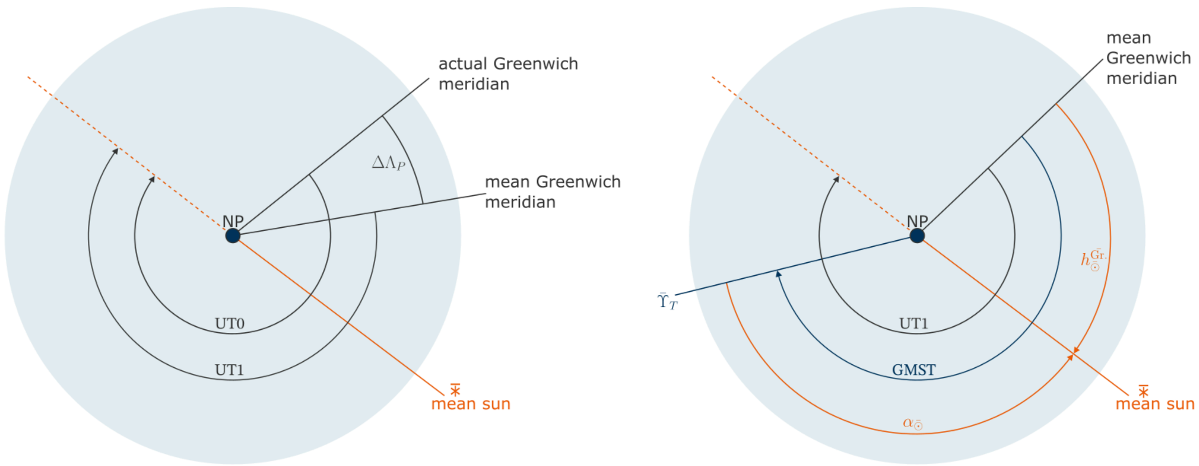

Figure 33 — Mean Greenwich meridian and UT1 (left) and conversion between solar and sidereal time (right)

8.3.4. Length Of Day

Still after these two refinement steps, UT1 will still not be completely stable. Due to mass redistributions on and in the Earth, the length of a mean solar day will not be constant. These fluctuations will become apparent after comparing every length of a solar day (LOD) to 86400 atomic seconds, cf. next section. The difference, or excess LOD is at the millisecond level.

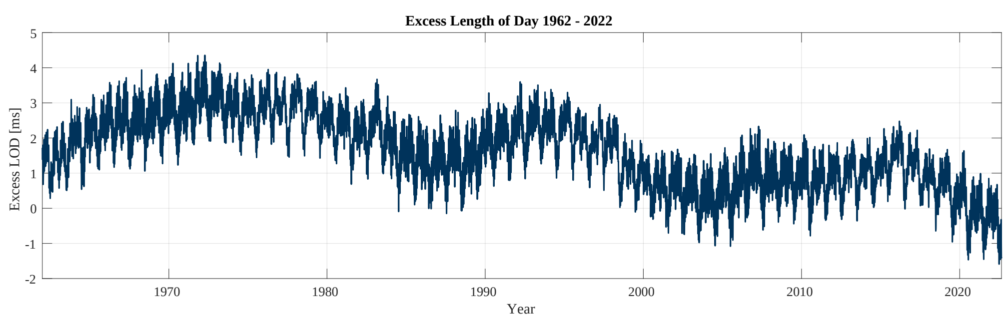

Figure 34 — Excess Length-Of-Day.

[**SOURCE:** IERS]

NOTEA further refinement is sometimes made in order to correct for predictable periodic length-of-day variations. The resulting solar time is called UT2.

8.3.5. Conversion between solar and sidereal times

The conversion between solar and sidereal time is straightforward. Figure 33 (right) shows the mean equinox, the mean solar meridian and the mean Greenwich meridian:

$$\mathrm{GMST} = \alpha_{\bar{\odot}} + h_{\bar{\odot}}^{\mathrm{Gr.}} = \alpha_{\bar{\odot}} + \mathrm{UT1} - 12^{\mathrm{h}} \quad (31)$$

The right ascension of the mean Sun is given by the internationally adopted formula (IAU 1976):

$$\alpha_{\bar{\odot}} = 18^{\mathrm{h}}41^{\mathrm{m}}50^{\mathrm{s}}.54{,}841 + 8{,}640{,}184.812866\,T + \mathrm{O}(T^2) \quad (32)$$

in which T is the time since the reference epoch J2000.0, counted in Julian centuries of 36525 days. Thus, $T = d/36{,}525$ with $d = \mathrm{JD} - 2{,}451{,}545.0$. The right ascension of the mean Sun is counted in seconds. Thus, the factor in the second term at the right-hand side has units of second per Julian century. Indeed, the number 8640184.812866 corresponds to one full circle ($= 24^{\mathrm{h}}$) per year or $3^{\mathrm{m}}56^{\mathrm{s}}.33$ per day. Therefore the conversion is appropriate:

$$\mathrm{GMST} = \mathrm{UT1} + 24{,}110.54841 + 8{,}640{,}184.812866\,T + \mathrm{O}(T^2) \quad (33)$$

Again, this formula is in seconds. Moreover, an integer times 24^{h} must be added in order to keep $\mathrm{GMST} \in 0;24^{\mathrm{h}}$.

8.4. Atomic time

The “second” was originally defined as the fraction $1/86\,400$ of a mean solar day. Since the Earth’s rotation shows irregularities at millisecond level in addition to a slowly decreasing spin rate, this definition became obsolete in the 20th century. With the advent of high precision frequency standards, based on atomic or molecular oscillations, the second was redefined in 1967:

> The second is the duration of 9192631770 periods of the radiation corresponding to the transition between the two hyperfine levels of the ground state of the cesium 133 atom.
>
> — 13th CGPM,

Moreover, clocks based on this atomic principle should be resting at sea level. According to relativity theory, a moving clock, or a clock that changes its potential energy level, will show a different time.

NOTEAtomic clocks on-board of GPS-satellites are corrected for these relativistic effects.

Nowadays, Cesium clocks achieve a stability below $100\ \mathrm{ps}$ per day with a relative precision of 10^{-15}. Such clocks lose a second after a period of approximate 30 million years. More recently this stability was improved by up to two orders of magnitude by *hydrogen masers* and *optical ion traps* which should be compared to the aforementioned astronomical definition, which may achieve a stability of $1\mathrm{ms}$ per day, or a relative precision of 10^{-8}.

Two main realizations of atomic time exist.

- **TAI**, which stands for The International Atomic Time (fr.: *Temps Atomique International*), is maintained by the Bureau International des Poids et Mesures (BIMP) from the readings of more than 200 atomic clocks located in metrology institutes and observatories in more than 30 countries around the world. The TAI is a weighted average of these clock readings.

- **GPS** system time is given by its *Composite Clock (CC)*. The CC or *paper* clock consists of all 5 monitor stations and satellite frequency standards. Since a different set of clocks is used for this realization of atomic time, T_{GPS} will be different from TAI. Apart from a constant offset of $19\,\mathrm{s}$, there will be small deviations up to μs level. These differences are broadcast in the navigation message as parameters A_0 and A_1.

8.4.1. UTC

The highly stable atomic TAI and the universal time UT1 will diverge over the years, as the daily LoD values accumulate. The difference UT1-TAI will increase in a non-homogeneous manner. This effect is caused by the slowing of the Earth's rotation and by irregularities in the spin rate.

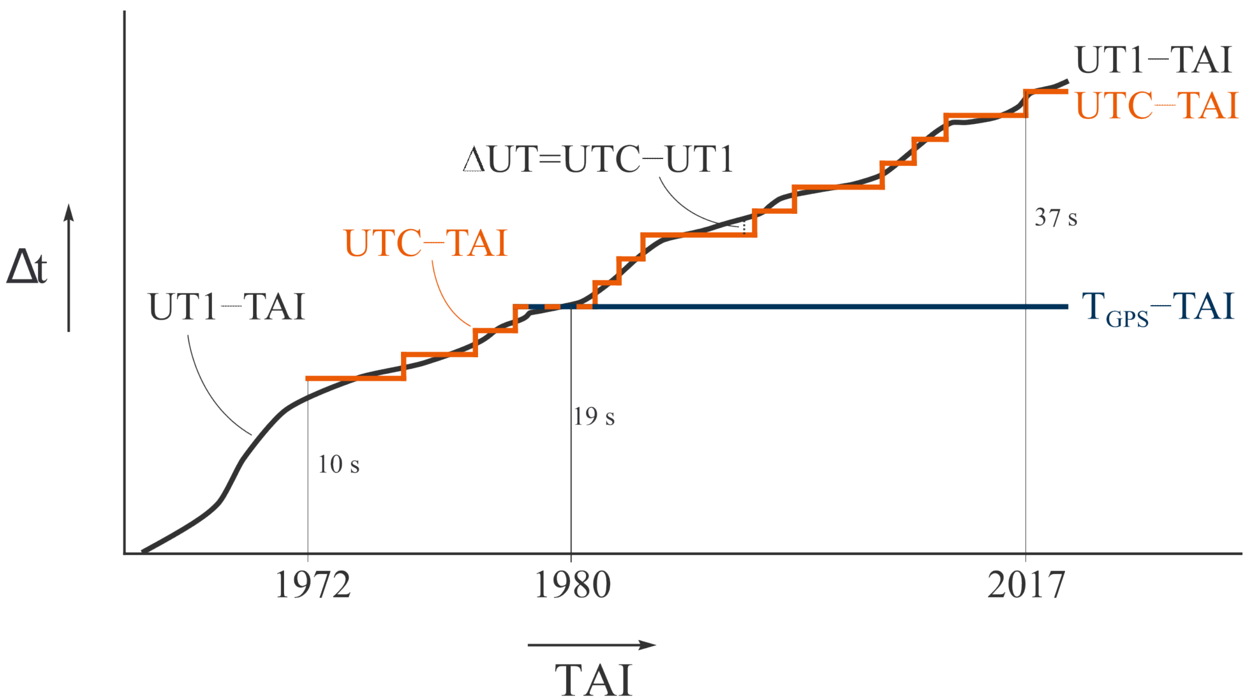

Figure 35 — Atomic time scales and UT1.

As a compromise — to keep atomic and solar time close to each other — *Universal Time Coordinated* (UTC) is introduced. The stable time scale of UTC is inherited from TAI. To stay close to UT1, leap seconds have been introduced such that the difference $\Delta UT = UTC - UT1 \leq 0.9\,\mathrm{s}$. The resulting time system meets all criteria, mentioned in the introduction:

1. stability: stable definition of the second,
2. accessibility: through atomic clocks and radio broadcast, and
3. meaningfulness: very close to mean solar time, i.e., the rotation phase of the Earth.

The corresponding formulas:

$$\Delta \mathrm{UT} = \mathrm{UTC} - \mathrm{UT1} \leq 0.9\,\mathrm{s} \quad (34)$$

Leap seconds are issued by the IERS, either on July 1 or January 1. The IERS makes a decision whenever $\Delta \mathrm{UT}$ comes close to the condition Formula (34). This system started in 1972 with $n = 10\,\mathrm{s}$. The last leap second was issued January 1, 2017, bringing the total to $n = 37\,\mathrm{s}$.

The CGPM published a statement in the Resolution 4 of the 27th November CGPM (2022) that until the year 2035 the maximum difference of UTC and UT1 of 0.9 seconds will be increased. This is the result of a long discussion since the introduction of leap seconds creates discontinuities that risk causing serious malfunctions in critical digital infrastructures including the Global Navigation Satellite Systems (GNSSs), telecommunications, and energy transmission systems.

At the start of GPS system time at midnight January 6, 1980, the difference between T_{GPS} and TAI was $19\,\mathrm{s}$. Leap seconds are neither applied to T_{GPS} nor to TAI. Consequently, UT1 and UTC are also drifting away from GPS time. Thus, T_{GPS} is now $13\,\mathrm{s}$ apart from UTC. In general:

$$T_{\mathrm{GPS}} - \mathrm{UTC} = N - f(A_0, A_1), \quad \text{with } N \in \mathrm{N} \text{ and } f(A_0, A_1) < 1\,\mu\,\mathrm{s} \quad (35)$$

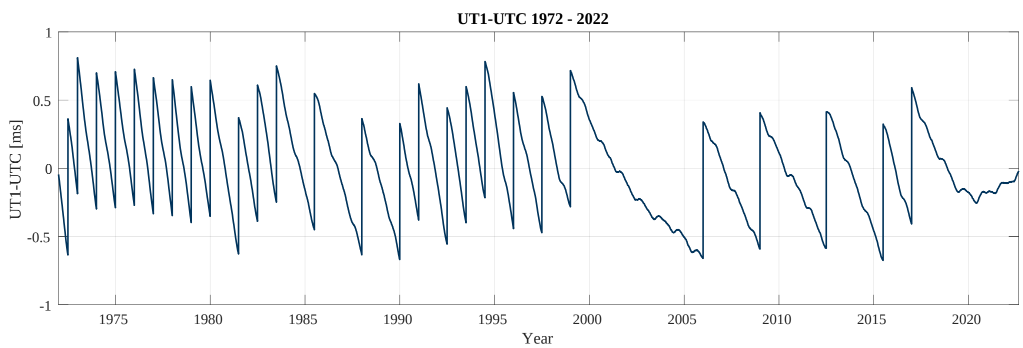

Figure 36 — Ut1-UTC since 1972.

[**SOURCE:** IERS]

8.5. Theoretical time

A problem with atomic clocks is relativistic effects, mainly due to the velocity v and gravitational potential U at the location of the clock. A clock is slower when it is in motion and under the influence of a gravitational potential. Theoretical time systems are often also referred to as dynamic time systems. In this ER the following theoretical time systems are discussed.

- **TCG** *Temps Coordonné Geocentrique* (geocentric coordinate time) corresponds to the time coordinate of a system that is referenced to the center of mass of the Earth. TCG uses the SI-second as unit.

- **TCB** *Temps Coordonné Barycentrique* (barycentric coordinate time) accordingly corresponds to the time coordinate of a system that is referenced to the barycenter. As with the TCG, the SI-second is used.
- **TDT / TT** *Temps Dynamique Terrestre* (dynamic terrestrial time) / *temps terrestre* (terrestrial time), which differs from TCG only by a scaling and on the geoid it agrees with $\mathrm{TAI} + 32.184\,\mathrm{s}$.
- **TDB / TB** *Temps Dynamique Barycentrique* (dynamic barycentric time) / *temps barycentrique* (barycentric time), which differs from TCB only by one scale, and on the geoid it coincides on average with TT.

The two geocentric times (TCG and TT) are most suitable for investigations in near-Earth space. On the other hand, they are good for the description of dynamics of the solar system and orbit calculations of interplanetary projects. The other time scale from the pair respectively differs from the coordinate time only by a linear mapping, which is chosen in such a way that it agrees with $\mathrm{TAI} + 32.184\,\mathrm{s}$ on Earth as exactly as possible or only with discrepancy of less than 2 ms.

8.5.1. Geocentric and barycentric coordinate time system

Since those two systems mainly vary in their reference point they can be discussed together. The zero point of both scales is set so that the event January 1, 1977, $0^{\mathrm{h}}0^{\mathrm{m}}0.000^{\mathrm{s}}$ TAI at the geocenter corresponds to the time January 1, 1977, $0^{\mathrm{h}}0^{\mathrm{m}}32.184^{\mathrm{s}}$ for both TCB and TCG. The measurement of a clock is called proper time τ and can be converted to the coordinates time systems t via:

$$\frac{d\tau}{\mathrm{dt}} = 1 - \frac{U}{c^2} - \frac{1}{2}\frac{(v)^2}{c^2} \tag{36}$$

with U being the gravitational potential. A stationary clock on the Earth has a fixed position r in a geocentric coordinate system, rotating at the angular velocity Ω (of the Earth's rotation) rotating in the inertial frame. The velocity of the clock in a non-rotating geocentric reference system is expressed as:

$v = \Omega \times r.$

Therefore, the proper time results in:

$$\frac{d\tau}{\mathrm{dt}_{\mathrm{TCG}}} = 1 - \frac{U_{\mathrm{geo}}}{c^2} \quad \text{with} \quad U_{\mathrm{geo}} = U + \frac{1}{2}(\Omega \times r)^2 \tag{37}$$

The geopotential U_{geo} additionally includes the potential created by centrifugal force. Therefore, clocks at locations with identical geopotential number (the numerical difference between two different equipotential surfaces) are equally fast in respect to TCG. Non-moving clocks on the geoid have equal speeds.

Transformation between TCB and TCG

For this transformation, the entire trajectory since T_0 (January 1977 $0^h0^m32.184^s$) must be known. Therefore, the calculation is very complex with high accuracy. A closed expression for TCG is:

$$\mathrm{TCG} = \mathrm{TCB} - \frac{1}{c^2}\int_{T_0}^{t}\left[U_{\mathrm{ext}}\, r_e(t') + \frac{v_e(t')^2}{2}\right]\mathrm{d}t' - \frac{1}{c^2}\left(v_e(t)\cdot(r - r_e(t)) + O\left(\frac{1}{c^4}\right)\right) \tag{38}$$

Here U_{ext} is the Newtonian gravitational potential generated by all bodies in the solar system except the Earth.

8.5.2. Terrestrial Time

The TCG is faster than the proper time measured by a clock. The Terrestrial Time TT is defined as a variation of the TCG, in which this higher rate is compensated:

$$\mathrm{TT} = \mathrm{TCG} - L_G(\mathrm{TCG} - T_0) \tag{39}$$

The discrepancy from a relative progress rate of 1 is $L_G = 6.969290134 \cdot 10^{-10} \approx 22\,\frac{\mathrm{ms}}{\mathrm{a}}$. The value of L_G corresponds to the gravitational time dilation $\frac{U_{\mathrm{geo,0}}}{c^2}$ for the geopotential $U_{\mathrm{geo,0}} = 62{,}636{,}856.0\frac{\mathrm{m}^2}{\mathrm{s}^2}$ which is the best value for the geopotential of the geoid known when the IAU-resolution was passed in 2000. Thus, while the definition of TT becomes independent of the intricacy and temporal changes inherent to the definition and realization of the geoid, the TT-second on the rotating geoid is, with very high accuracy, the SI-second and TT is further approximated very well by $\mathrm{TAI} + 32.184\,\mathrm{s}$.

8.5.3. Barycentric time

The barycentric time was originally intended to serve as an independent time argument of barycentric ephemerides and equations of motion. The transformation between TCB and BT is similar to the transformation between TT and TCG:

$$\mathrm{TDB} = \mathrm{TCB} - 65.5\mu\mathrm{s} - L_B(\mathrm{TCB} - T_0) \tag{40}$$

with the same T_0 as in TT and $L_B = 1.550519768 \cdot 10^{-8} \approx 0.49\,\frac{\mathrm{s}}{\mathrm{a}}$.
The value of L_B was chosen so that TDB and TT have the same average progress rate at the center of mass of the Earth, so the difference remains limited.

8.6. Calendar time

The UTC is the basis of standard time, in which the Earth is roughly divided in 24 meridional zones extending 15° in longitude. Each individual zone has one single civilian time system, which differs an integer number of hours from UTC. This guarantees that civilian time is never off by more than half an hour compared to true local time. The situation is slightly more complex, depending on local geography and state boundaries. Some zones even have a non-integer difference ΔZ to UTC:

$$\text{standard time: } T_Z = \text{UTC} + \Delta Z \quad (41)$$

8.6.1. Julian Days

Civilian time is counted in the *Gregorian Calendar* in days (D), months (M), and years (Y). Since the length of months is variable and since some years (leap years) have an additional day, it is difficult to calculate time intervals in terms of days. For geodetic, astronomic, and chronological purposes a chronological counting of days would be more practical, which is what Julian Day Numbers are.

The Julian Day system begins at noon on January 1, -4712. A new Julian Day starts at noon. This makes sense for astronomical purposes (hour angle equals zero). Many algorithms to compute Julian Day numbers from calendar dates exist. For instance:

$$\text{JD} = 367\,Y - \text{floor}(7(Y + \text{floor}((M+9)/12))/4) + \text{floor}(275\,M/9) + D + 1{,}721{,}014 + \text{UT1}/24 - 0.5 \quad (42)$$

$$\text{MJD} = \text{JD} - 2{,}400{,}000.5 \quad (43)$$

The latter is the *Modified Julian Day*, which starts at midnight November 16-17, 1858. It guarantees that at most 5 digits are required in the period from 1859 to about 2130. The 0.5 at the end means that a new MJD starts at midnight.

[**SOURCE:** Astronomical Algorithms]

9

REPRESENTATION OF SPACE OBJECTS IN VARIOUS COORDINATE SYSTEMS

9 REPRESENTATION OF SPACE OBJECTS IN VARIOUS COORDINATE SYSTEMS

This chapter of the report is intended to describe different ways of representing features in space and show their differences.
In space geodesy, the objects of interest are often satellites orbiting the Earth. For a simple description of this motion a Kepler ellipse is usually approximated, which can be described by the six Keplerian elements. The number six is equal to the sum of three Cartesian position coordinates and three velocity components.

9.1. Keplerian elements

The Keplerian elements can essentially be divided into three categories. The elements are stated as follows.

a Semi-major axis [meter]

e Eccentricity

I Inclination [degree]

Ω Right ascension of the ascending node [degree]

ω Argument of perigee [degree]

n Mean motion [rad/seconds]

9.1.1. Size and shape of ellipse

First, the ellipse itself must be described. Figure 37 shows the basic elements of an ellipse. In the case of a satellite orbiting the Earth, the focus would be the center of mass of the Earth and the satellite follows the outline of the drawn ellipse. When orbiting around the Earth, the terms *perigee* and *apogee* are used for the peri- and apofocuses. The apofocus is the point on an elliptic orbit at the greatest distance from the principal focus. The (first) *eccentricity* e is defined by the *semi-major axis* a and the *semi-minor axis* b:

$$e = \sqrt{\frac{a^2 - b^2}{a^2}} \quad (44)$$

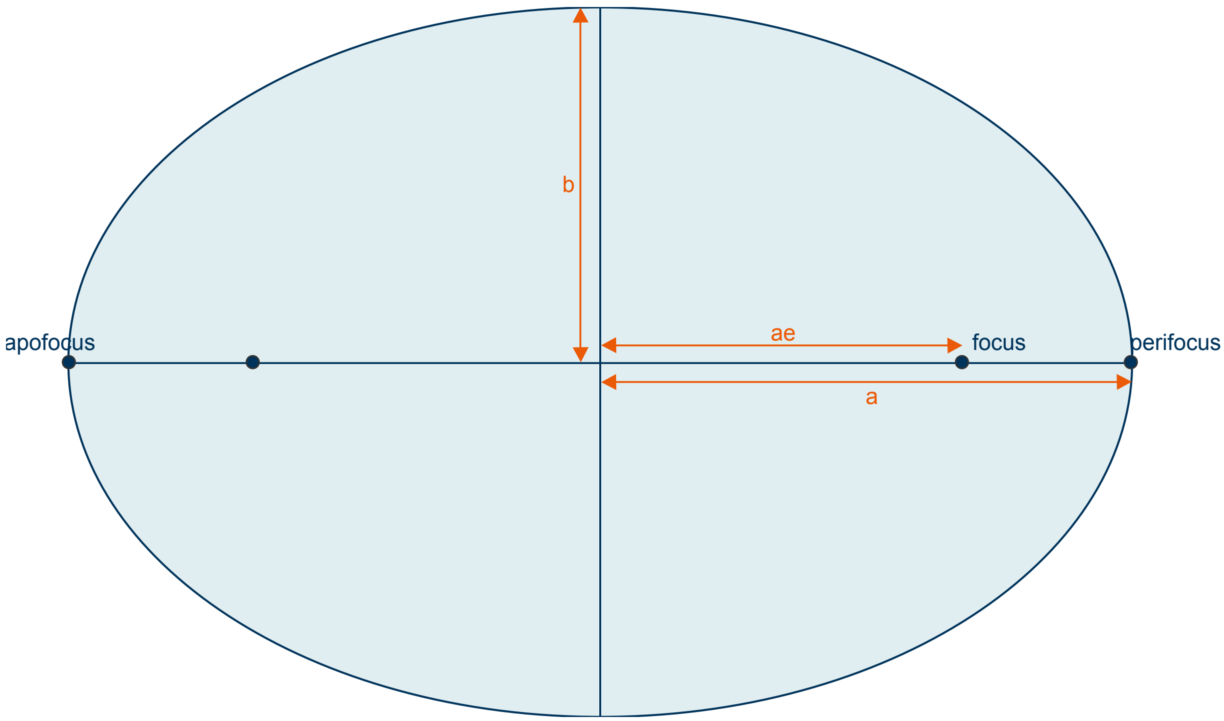

Figure 37 — Basic elements of an ellipse

9.1.2. Orientation of orbital plane in space

The coordinate axes are as defined in the conventional inertial reference system. The orbital plane is tilted with respect to the equator as seen in Figure 38. The corresponding angle I is called *Inclination*. The intersection line between orbital and equator plane is the nodal line. The node, in which the satellite crosses the equator from south to north, is the ascending node. The angle in the equatorial plane between the vernal equinox ($\Upsilon\#$) and ascending node is the *right ascension of the ascending node* Ω. The angle within the plane between the equator plane and the perigee is called *argument of perigee* ω.

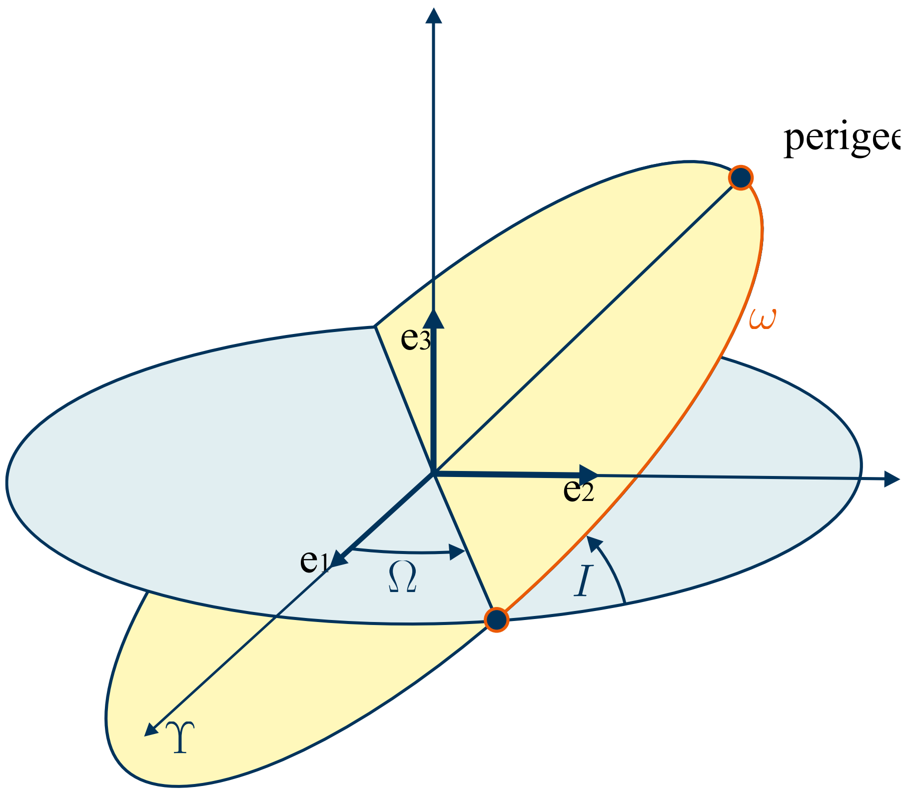

Figure 38 — Orientation of orbital plane in space.

9.1.3. Position within orbital plane

A simple way to define the position within the orbital plane is to refer to the time that passes during one full revolution, which is called *orbit period* T. The commonly used alternative is the *mean motion* n that describes the mean angular velocity within the orbital plane.

$$n = 2\frac{\pi}{T} = \sqrt{G\frac{M}{a^3}} \quad (45)$$

NOTEThere are several sources for the download of Kepler elements of various satellite missions, the best known being the so-called Two Line Elements (TLE) from NORAD (North American Aerospace Defense Command) and NASA.

9.2. Conventional inertial reference system

Using the Keplerian elements, two different features are shown in various reference systems. The two objects of interest are the International Space Station (ISS) and one of the satellites of the Galileo-constellation (GSAT0101). The according Two Line Elements were used to calculate 5 full revolutions in October 2022. In addition to the Kepler ellipse the influence of the flattening of the Earth is considered. To make the representation more descriptive, a

geocentric coordinate system was chosen. When looking at an interplanetary orbit, for example, this translation has to be considered.

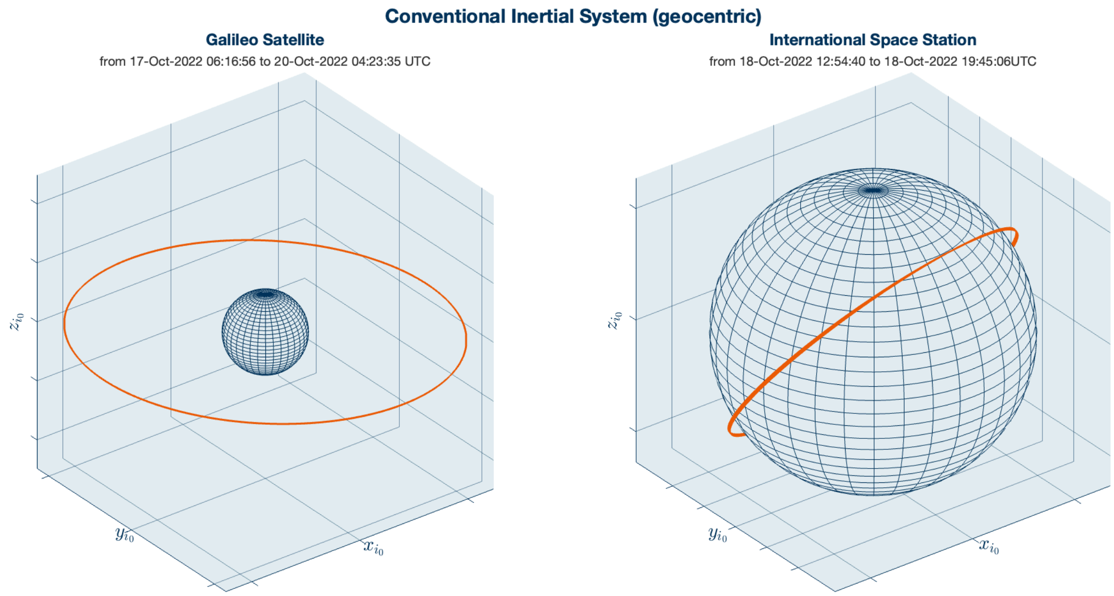

Figure 39 — 5 revolutions of GSAT101 and ISS in geocentric conventional inertial reference system.

9.3. Conventional terrestrial reference system

Using the formulas for the transformation as given in Clause 7 the coordinates of the trajectory are transformed into a conventional terrestrial reference system. The easiest presentation of those coordinates is the 3D-plot of the trajectory. Additionally, the ground track is shown after using a transformation to ellipsoidal coordinates and projecting those. A Miller-projection is used for the representation as a 2D map.

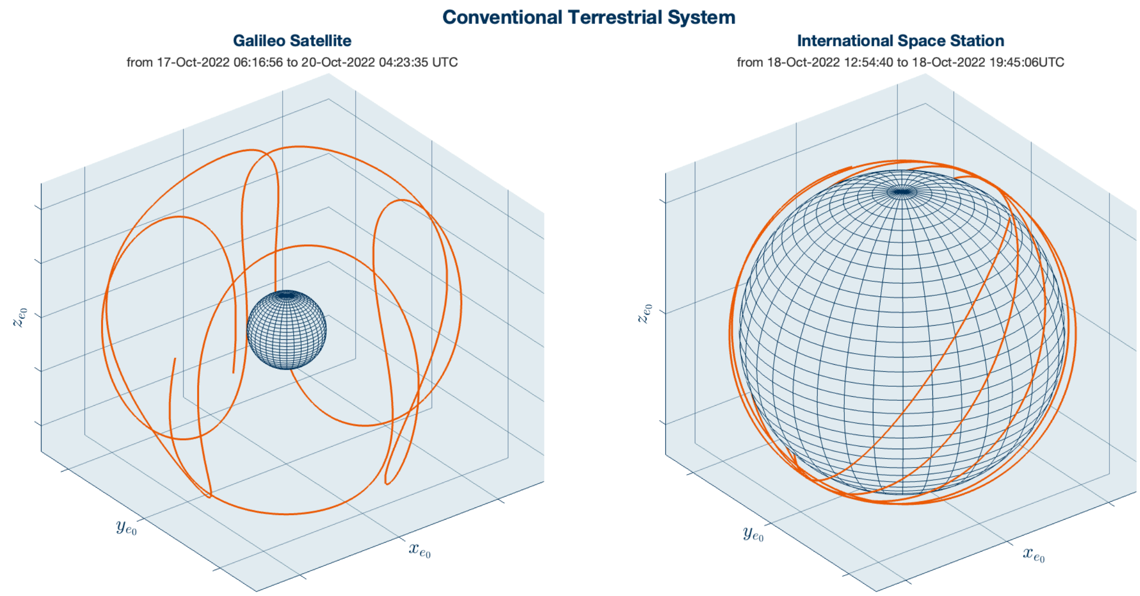

Figure 40 — 5 revolutions of GSAT101 and ISS in geocentric conventional terrestrial reference system.

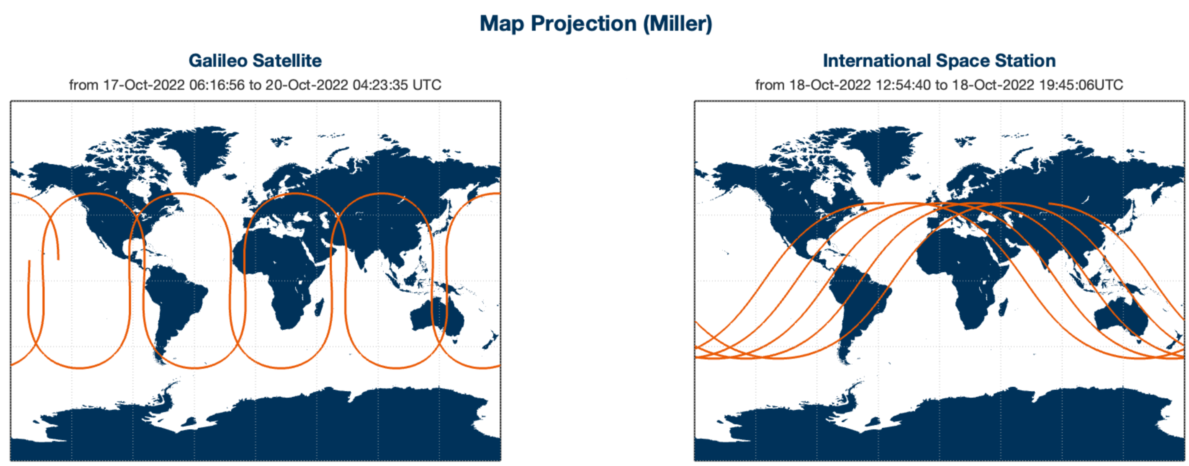

Figure 41 — Ground track of 5 revolutions of GSAT101 and ISS in map projection.

9.4. Local system: Skyplot

The trajectory is transformed into a local system. Plotting cartesian coordinates of a world-wide trajectory would be more confusing than helpful. The origin of the local system was chosen to be at the Campus of the University of Stuttgart, Germany. Therefore, a so-called skyplot was created, which calculates the azimuth and elevation for each point of the trajectory. The elevation describes the angle between the local horizon and the object, the azimuth, and the angle between the object and the north pole within the horizon plane.

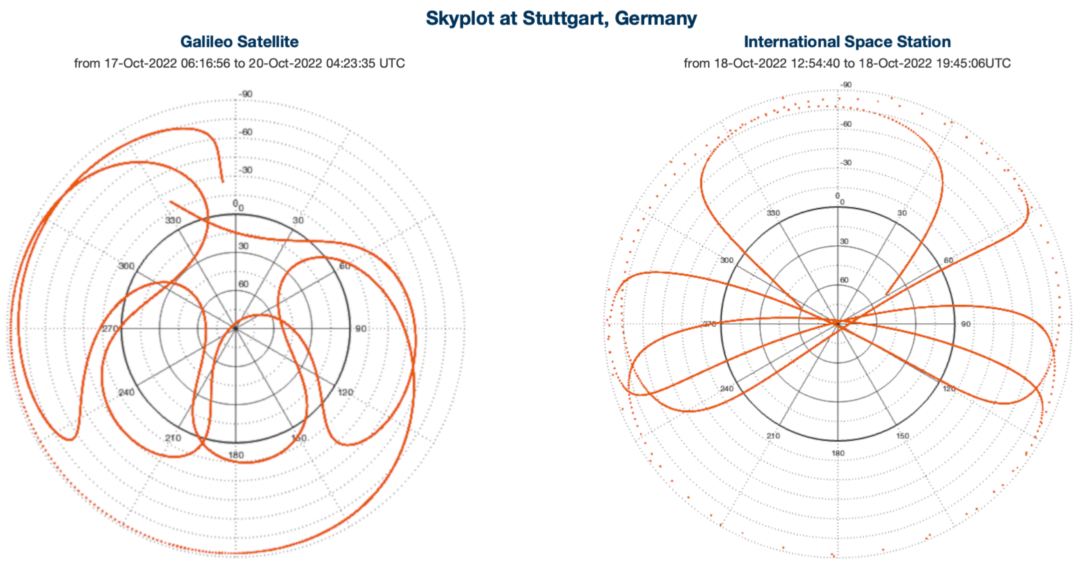

Figure 42 — Skyplot of 5 revolutions of GSAT101 and ISS

10

EVALUATION OF EXISTING STANDARDS FROM GEODETIC POINT OF VIEW

10 EVALUATION OF EXISTING STANDARDS FROM GEODETIC POINT OF VIEW

A variety of coordinate reference systems have been defined for use around the world or in specific regions and for different purposes. These coordinate systems are a critical foundation for geoinformatics sciences and technologies, including cartography, geographic information systems, surveying, remote sensing, and civil engineering. This has led to the development of a model and schema for describing exchanging information for coordinate and coordinate reference systems: ISO 19111:2019 *Geographic information-Spatial — referencing by coordinates*. This international standard is the basis for a number of CRS registries, such as EPSG. The following chapter evaluates existing Standards as described in Clause 5 and elaborates weaknesses and deficits from a geodetic point of view. To be precise, the benchmark, from a geodetic point of view, is the definitions and conventions as defined by the International Earth Rotation and Reference Systems Service (IERS) discussed in Clause 6 of this Engineering Report. Such a benchmark would be valuable as IERS processes all geodetic measurements to artificial Earth satellites and quasars to calculate the position of the Earth in the astronomical fundamental system. This is a necessary base for the exact positioning of sensors in 3D space and corresponding 3D data streaming, analytics, and portrayal and plays an important role in many geospatial scenarios and applications. Since this ER was developed in the context of an OGC testbed, special attention is paid to OGC and ISO Standards.

10.1. Definition of an inertial system

ISO 19111 allows the definition of various coordinate systems that are used in many applications. Most of the OGC Member organizations' data are typically measured in an Earth-based system such as WGS84 or some type of local coordinate system. However, these coordinate systems are generally not adequate for all types of data and not suitable for representing all space objects. In geodesy or aerospace it is often not possible to represent everything with an Earth-based reference system. When considering an object that is (approximately) independent of the Earth and its gravity, it becomes complicated when it is specified in a coordinate reference system that is tied to the Earth in all its properties. A **non-moving** and **non-rotating** coordinate reference system is needed, namely an inertial frame.

Please note that all "inertial" reference frames are only approximations. The simplest way to define such an approximation is to determine its transformation to an Earth-fixed system. The simplest possible transformation would use a constant rate of rotation of the Earth and no variation of any of its axes. A more complex version is described in the Clause 6 chapter, which takes into account effects such as precession, nutation, and polar motion.

In developing such a coordinate system, one would need to choose one of the systems stated in ISO 19111: *geodetic*, *engineering*, *vertical*, *parametric*, or *temporal*. A geodetic coordinate system is Earth-fixed (exceptions are possible but unintuitive), since it describes spatial data on or near the surface of the Earth. Parametric, vertical, and temporal systems are not applicable to the problem, so currently within ISO 19111 an inertial coordinate system can be characterized as the engineering subtype. Considering time and in combination with the temporal subtype, it can be defined as the so-called spatio-parametric-temporal compound coordinate reference system. However, by definition, these are designed for use in small areas, spatial location on moving objects, or spatial location inside an image.

Therefore, there is no subtype of coordinate reference system matching the requirements for inertial reference systems. To this end, definition of an inertial coordinate system, which is necessary for representing space objects, does not fit into the current Standard as provided by ISO or OGC. The *Navigation Data* conventions of the CCSDS also doesn't address the subject of the definition of an inertial system but refers to the IERS. NASA SPICE has a definition of an inertial system, but this is not equivalent to the IERS. In SPICE, an inertial system must always originate in the barycenter of the solar system. Whether this restriction is useful for a generally usable definition is questionable but would have to be discussed.

Solution
The simplest way to solve this problem would be to consider inertial systems as allowable subtypes of coordinate systems in the ISO 19111 model and schema. An "inertial" subtype would deal with the description of all objects in space. A general definition for an inertial reference system would need to be developed based on the definitions and conventions provided by IERS.

10.2. Definition of barycentric celestial reference systems in ISO / OGC

The barycentric celestial reference system (BCRS) is a coordinate system used in astrometry to determine the location and motions of astronomical objects. BCRS was created in 2000 by the International Astronomical Union (IAU) as a global Standard reference system for objects located outside the gravitational neighborhood of planets, moons and other solar system bodies, stars and other objects in the Milky Way Galaxy, and extragalactic objects.

ISO 19111 allows for coordinate reference systems to define their origin and orientation using other objects than the Earth for planetary use. The definition of a barycentric coordinate reference system is therefore possible, but not very intuitive and pleasant. Additionally, other planets could be used for the definition of a coordinate reference system.

Solution
Since the coordinate system required is very close to the geodetic subtype, a solution would be to rename it "planetodetic" as is the case in NASA SPICE. Other suggestions, such as adding "barycentric", "marscentric", etc. to the catalog, would complicate things unnecessarily. A organization with great expertise in this topic is the IAU Working Group for Cartographic Coordinates & Rotational Elements.

10.3. Definition of time systems in ISO / OGC

Time can be defined as a coordinate in a fourth dimension. The access to time units is easy, because many of the observed changes are periodic. If the changing phenomenon varies with uniform period, then the associated time scale is also uniform. Obviously, a desirable property of a description and realization of time is that its scale be uniform, at least on a local scale. However, only very few observed dynamical systems have strictly uniform time units. In the past, the Earth's rotation represented the most appropriate and obvious phenomenon to represent the time scale, with the unit being a (solar) day. However, as described in Clause 8 it has long been known that the Earth's rotation is not uniform. Therefore, a theoretical uniform time needs to be defined to represent the fourth dimension. The need for a uniform time scale for representing extraterrestrial objects must be clarified in the ISO 19111 Standard.

In addition to the scale or units, an origin must be defined for each Coordinate Reference System (CRS) for which a specific time system must also be specified, such as a zero point or epoch at which a time value is established. This also needs to be clarified within the ISO 19111.

Solution
Current ISO Standards should be revised so that the time component is relevant for all features and should always be considered, especially for aerospace applications. Within the ISO 19111, to account for all possible time systems required for different applications, a viable solution would be to define different sub-subtype categories within the temporal CRS, similar to the outline of Clause 8.

A

ANNEX A (INFORMATIVE) CURRENT STANDARDS

A

ANNEX A (INFORMATIVE)

A.1. GeoPose

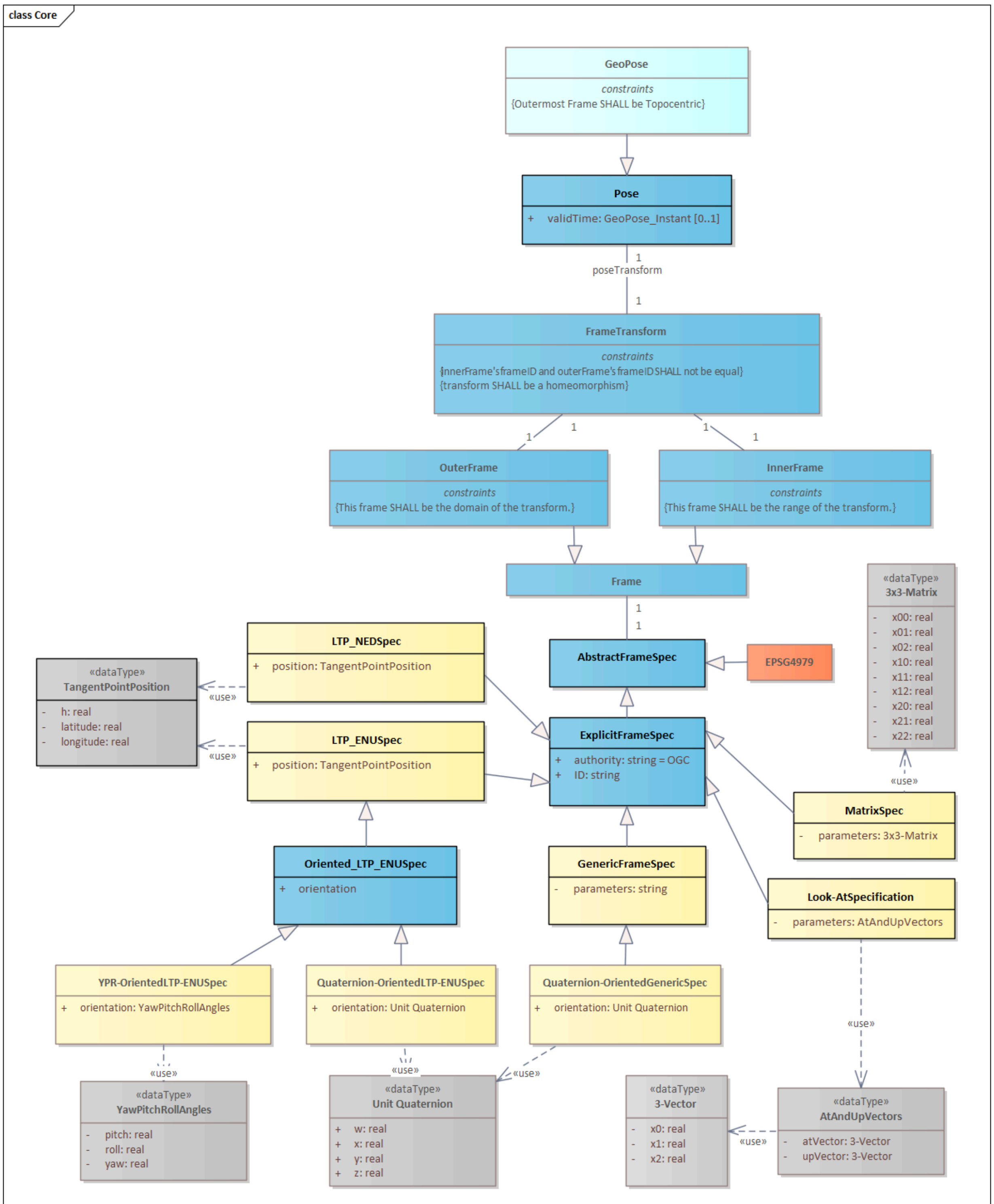

Figure A.1 — GeoPose Core

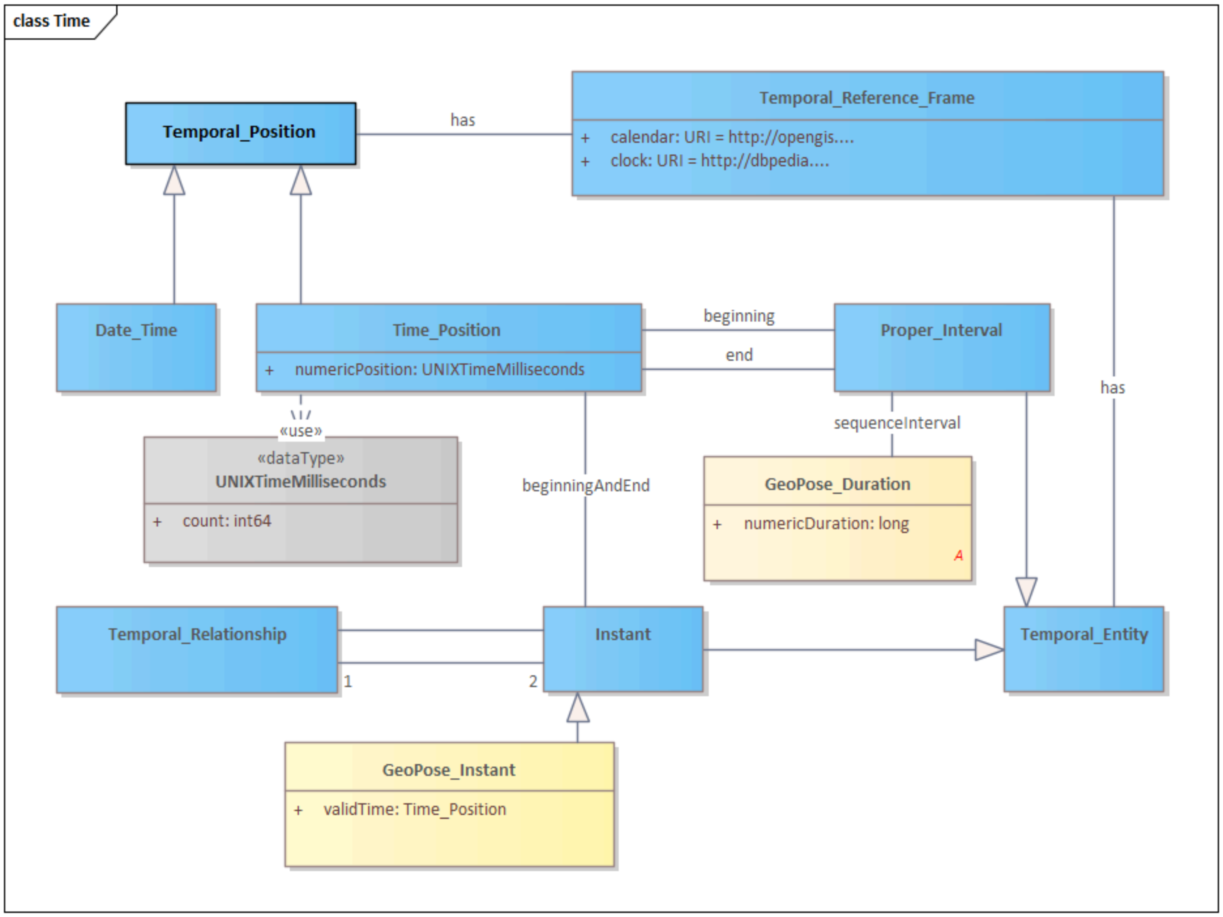

Figure A.2 — GeoPose Time Model

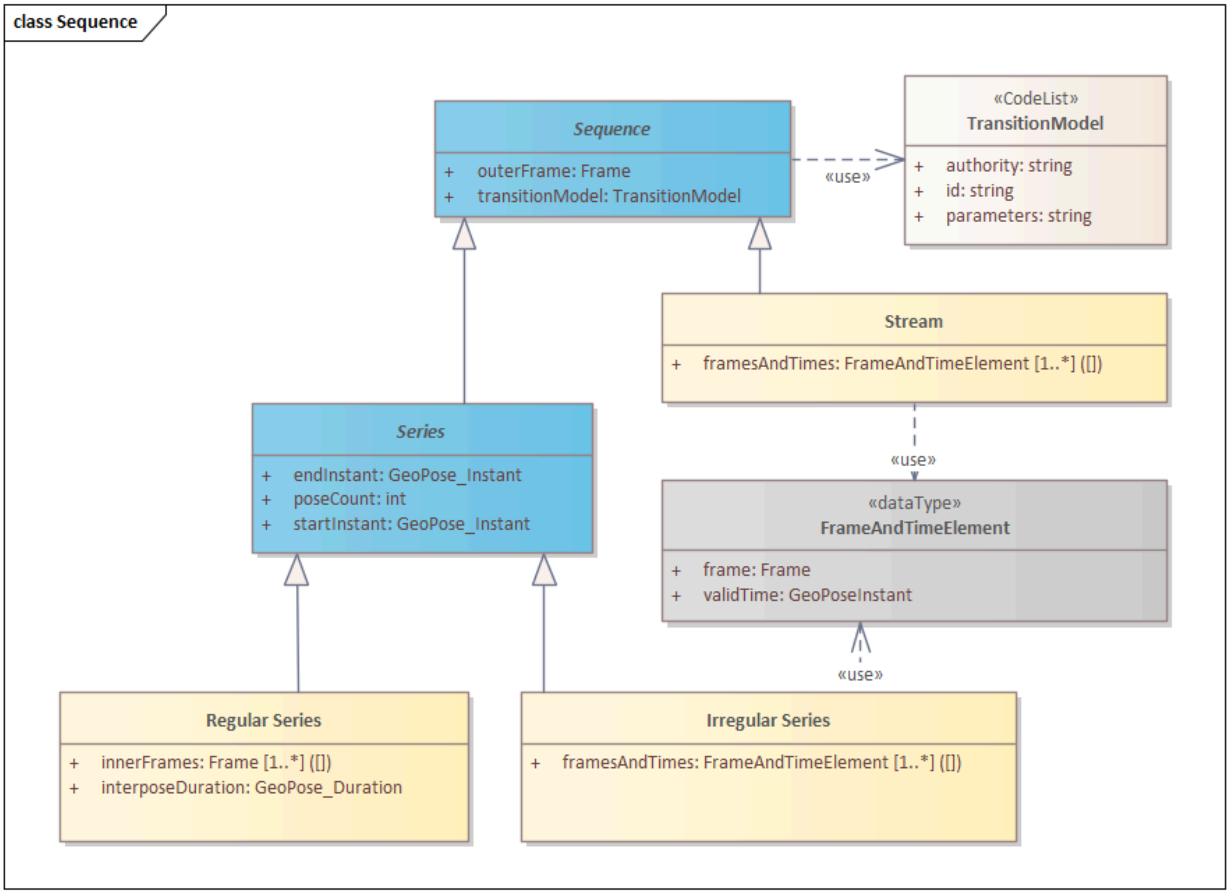

Figure A.3 — GeoPose Sequence Model

B

ANNEX B (INFORMATIVE) REVISION HISTORY

B

ANNEX B (INFORMATIVE) REVISION HISTORY

DATE	RELEASE	AUTHOR	PRIMARY CLAUSES MODIFIED	DESCRIPTION
2016-04-28	0.1	G. Editor	all	initial version

BIBLIOGRAPHY

BIBLIOGRAPHY

[1] Akinori Asahara, Ryosuke Shibasaki, Nobuhiro Ishimaru, David Burggraf: OGC 14-084r2, *OGC® Moving Features Encoding Extension: Simple Comma Separated Values (CSV)*. Open Geospatial Consortium (2015). https://docs.ogc.org/is/14-084r2/14-084r2.html.

[2] Akinori Asahara, Ryosuke Shibasaki, Nobuhiro Ishimaru, David Burggraf: OGC 14-083r2, *OGC® Moving Features Encoding Part I: XML Core*. Open Geospatial Consortium (2015). https://docs.ogc.org/is/14-083r2/14-083r2.html.

[3] OGC: OGC 22-038: Reference Frame Transformation Engineering Report, 2022. http://ogc.pages.ogc.org/T18-3Dplus_Data_Standards_and_Streaming/documents/D025/document.html

[4] Astronomical Algorithms, Jean Meeus, https://www.agopax.it/Libri_astronomia/pdf/Astronomical%20Algorithms.pdf

[5] Consultative Committee for Space Data Systems, NAVIGATION DATA—DEFINITIONS AND CONVENTIONS,2019, https://public.ccsds.org/Pubs/500x0g4.pdf

[6] Springer Handbook of Global Navigation Satellite Systems, Peter J.G. Teunissen, Oliver Montenbruck, https://doi.org/10.1007/978-3-319-42928-1

[7] IERS Earth rotation data — https://www.iers.org/IERS/EN/DataProducts/EarthOrientationData/eop.html

[8] NASA NAIF, SPICE Documentation, retrieved January 2023, https://naif.jpl.nasa.gov/naif/tutorials.html

[9] The Open Group, UNIX Time, retrieved January 2023, https://pubs.opengroup.org/onlinepubs/9699919799/basedefs/V1_chap04.html#tag_04_16